This video will teach us how to expand a function according to the Maclaurin series. The general equation of the Taylor series is also presented.

Some tips for the stability equations regarding the total potential energy are also covered.

$y = f(x)$

Taylor : $f(x) = \sum \frac{f^{n}(a)}{n!} \cdot (x-a)^n$

$f(x) = x^2 + x + 1$

$a = 1$

$$\begin{cases} f(x) = x^2 + x + 1 & f(1) = 3 \\ f'(x) = 2x + 1 & f'(1) = 3 \\ f''(x) = 2 & f''(1) = 2 \\ f'''(x) = 0 & f'''(1) = 0 \end{cases}$$

$$f(x) = \frac{f^{0}(1)}{0!} \cdot (x-1)^0 + \frac{f'(1)}{1!}(x-1)^1 + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 + 0$$

$$f(x) = \frac{3}{1}(1) + \frac{3}{1}(x-1)^1 + \frac{2}{2}(x-1)^2 + 0$$

$$= 3 + 3x - 3 + x^2 - 2x + 1 = x^2 + x + 1$$

$a = 0 \Longrightarrow$ M.L series :

$$f(x) = \frac{f(0)}{0!}(x-0)^0 + \frac{f'(0)}{1!}(x-0)^1 + \frac{f''(0)}{2!}(x-0)^2 + \frac{f'''(0)}{3!}(x-0)^3 + \cdots$$

$$f(x) = f(0) + f'(0) \cdot x + \frac{f''(0)}{2!} \cdot x^2 + \frac{f'''(0)}{3!} x^3 + \cdots$$

Example:

$f(x) = \sin x$ $\qquad f(0) = 0$

$f'(x) = \cos x$ $\qquad f'(0) = 1$

$f''(x) = -\sin x$ $\qquad f''(0) = 0$

$f'''(x) = -\cos x$ $\qquad f'''(0) = -1$

$\vdots$ $\qquad \vdots$

$$\sin x = 0 + 1 \cdot x + 0\, x^2 + \frac{-1}{6} \cdot x^3 + \cdots$$

$$\sin x = x - \frac{1}{6}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \cdots$$

$y = \cos x$

$f(x) = \cos x \qquad f(0) = 1$

$f'(x) = -\sin x \qquad f'(0) = 0$

$f''(x) = -\cos x \qquad f''(0) = -1$

$f'''(x) = \sin x \qquad f'''(0) = 0$

$$\cos x = 1 + 0\frac{x^1}{1!} - 1\frac{x^2}{2!} + 0\frac{x^3}{3!} + \frac{1}{4!}x^4$$

$$\cos x = 1 - \frac{x^2}{2} + \frac{1}{4!}x^4 - \cdots$$

Example:

$f(x) = \sqrt{1+x}$

$f(x) = \sqrt{1+x} = (1+x)^{\frac{1}{2}} \qquad f(0) = 1$

$f'(x) = \frac{1}{2}(1+x)^{-\frac{1}{2}} \qquad f'(0) = \frac{1}{2}$

$f''(x) = -\frac{1}{4}(1+x)^{-\frac{3}{2}} \qquad f''(0) = -\frac{1}{4}$

$f'''(x) = \frac{3}{8}(1+x)^{-\frac{5}{2}} \qquad f'''(0) = \frac{3}{8}$

$$f(x) = f(0) + f'(0).x + \frac{f''(0)}{2!}.x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1/4}{2!}x^2 + \frac{3/8}{3!}x^3 - \cdots$$

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 - \cdots$$

$$\sqrt{1+x^2} = 1 + \frac{1}{2}(x)^2 - \frac{1}{8}(x^2)^2 + \frac{1}{16}(x^2)^3 - \cdots$$

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1/4}{2!}x^2 + \frac{3/8}{3!}x^3 - \cdots$$

$$\sqrt{1-x} = \sqrt{1+(-x)} = 1 + \frac{1}{2}(-x) - \frac{1}{8}(-x)^2 + \frac{1}{16}(-x)^3 - \cdots$$

$$\sqrt{1-x^2} = \sqrt{1+(-x^2)} = 1 + \frac{1}{2}(-x^2) - \frac{1}{8}(-x^2)^2 + \frac{1}{16}(-x^2)^3 - \cdots$$

$$\sqrt{1-x^2} = 1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 - \frac{1}{16}x^6 - \cdots$$

$$\sin x \underset{x\to 0}{\simeq} x \quad , \quad \cos x \underset{x\to 0}{\simeq} 1 - \frac{x^2}{2}$$

$$\sqrt{1+x} \underset{x\to 0}{\simeq} 1 + \frac{1}{2}x$$