According to Eurocode 1992-1-1:

a) Determine the definition of a concrete beam.
b) What assumptions are taken for a concrete beam?
c) What is the relationship between the stress and strain in concrete and steel?
d) What is balance reinforcement?
e) Formulate the bending moment calculation in a concrete beam.

PS. Neglect the effect of compressive steel.

5.3 Idealisation of the structure

5.3.1 Structural models for overall analysis

(1)P The elements of a structure are classified, by consideration of their nature and function, as beams, columns, slabs, walls, plates, arches, shells etc. Rules are provided for the analysis of the commoner of these elements and of structures consisting of combinations of these elements.

(2) For buildings the following provisions (3) to (7) are applicable:

(3) A beam is a member for which the span is not less than 3 times the overall section depth. Otherwise it should be considered as a deep beam.

(4) A slab is a member for which the minimum panel dimension is not less than 5 times the overall slab thickness.

(5) A slab subjected to dominantly uniformly distributed loads may be considered to be one-way spanning if either:
- it possesses two free (unsupported) and sensibly parallel edges, or
- it is the central part of a sensibly rectangular slab supported on four edges with a ratio of the longer to shorter span greater than 2.

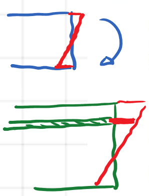
$L > 3h \rightarrow \boxed{h < \frac{L}{3}}$

SECTION 6 ULTIMATE LIMIT STATES (ULS)

6.1 Bending with or without axial force

(1)P This section applies to undisturbed regions of beams, slabs and similar types of members for which sections remain approximately plane before and after loading. The discontinuity regions of beams and other members in which plane sections do not remain plane may be designed and detailed according to 6.5.

(2)P When determining the ultimate moment resistance of reinforced or prestressed concrete cross-sections, the following assumptions are made:



- plane sections remain plane.
- the strain in bonded reinforcement or bonded prestressing tendons, whether in tension or in compression, is the same as that in the surrounding concrete.
- the tensile strength of the concrete is ignored.
- the stresses in the concrete in compression are derived from the design stress/strain relationship given in 3.1.7.
- the stresses in the reinforcing or prestressing steel are derived from the design curves in 3.2 (Figure 3.8) and 3.3 (Figure 3.10).
- the initial strain in prestressing tendons is taken into account when assessing the stresses in the tendons.

3.1.7 Stress-strain relations for the design of cross-sections

(3) A rectangular stress distribution (as given in Figure 3.5) may be assumed. The factor λ, defining the effective height of the compression zone and the factor η, defining the effective strength, follow from:

$$\lambda = 0{,}8 \quad \text{for } f_{ck} \leq 50 \text{ MPa} \tag{3.19}$$

$$\lambda = 0{,}8 - (f_{ck} - 50)/400 \quad \text{for } 50 < f_{ck} \leq 90 \text{ MPa} \tag{3.20}$$

and

$$\eta = 1{,}0 \quad \text{for } f_{ck} \leq 50 \text{ MPa} \tag{3.21}$$

$$\eta = 1{,}0 - (f_{ck} - 50)/200 \quad \text{for } 50 < f_{ck} \leq 90 \text{ MPa} \tag{3.22}$$

Note: If the width of the compression zone decreases in the direction of the extreme compression fibre, the value ηf_{cd} should be reduced by 10%.

compressive zone

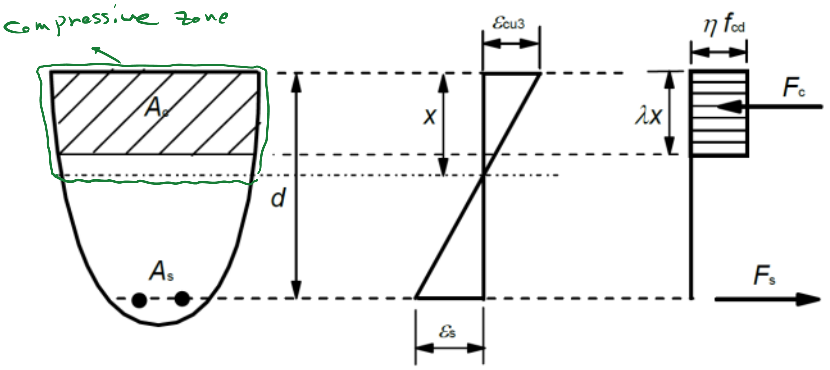


Figure 3.5: Rectangular stress distribution

5.6 Plastic analysis

5.6.1 General

(1)P Methods based on plastic analysis shall only be used for the check at ULS.

3.1.7 Stress-strain relations for the design of cross-sections

(2) Other simplified stress-strain relationships may be used if equivalent to or more conservative than the one defined in (1), for instance bi-linear according to Figure 3.4 (compressive stress and shortening strain shown as absolute values) with values of ε_{c3} and ε_{cu3} according to Table 3.1.

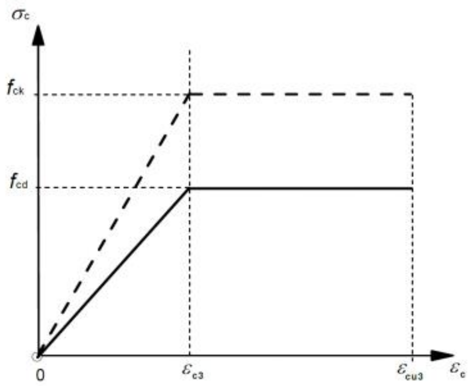


Figure 3.4: Bi-linear stress-strain relation.

3.1.6 Design compressive and tensile strengths

(1)P The value of the design compressive strength is defined as

$$f_{cd} = \alpha_{cc} f_{ck} / \gamma_C \qquad (3.15)$$

where:

γ_C is the partial safety factor for concrete, see 2.4.2.4, and

α_{cc} is the coefficient taking account of long term effects on the compressive strength and of unfavourable effects resulting from the way the load is applied.

Note: The value of α_{cc} for use in a Country should lie between 0,8 and 1,0 and may be found in its National Annex. The recommended value is 1.

Design compressive and tensile strengths

3.1.6(1)P

A value of 0.85 is adopted for the factor α_{cc}.

2.4.2.4 Partial factors for materials

(1) Partial factors for materials for ultimate limit states, γ_C and γ_S should be used.

Note: The values of γ_C and γ_S for use in a Country may be found in its National Annex. The recommended values for 'persistent & transient' and 'accidental, design situations are given in Table 2.1N. These are not valid for fire design for which reference should be made to EN 1992-1-2.

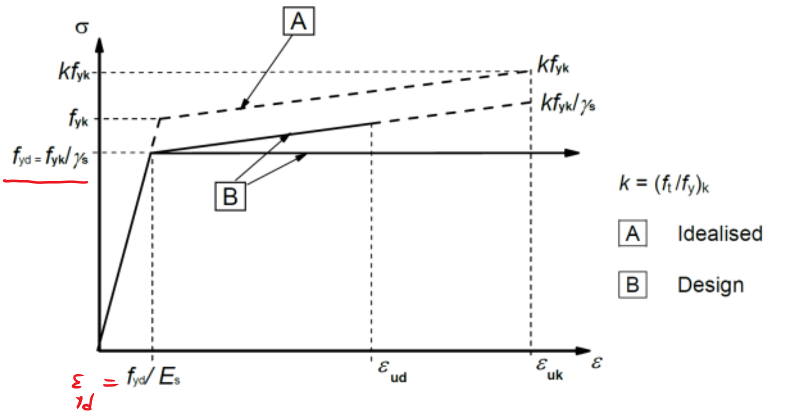
For fatigue verification the partial factors for persistent design situations given in Table 2.1N are recommended for the values of $\gamma_{C,fat}$ and $\gamma_{S,fat}$.

Table 2.1N: Partial factors for materials for ultimate limit states

Design situations	γ_C for concrete	γ_S for reinforcing steel	γ_S for prestressing steel
Persistent & Transient	1,5	1,15	1,15
Accidental	1,2	1,0	1,0

3.2 Reinforcing steel

3.2.7 Design assumptions



Class B
Class C

$f_{yk} = 500\ MPa$

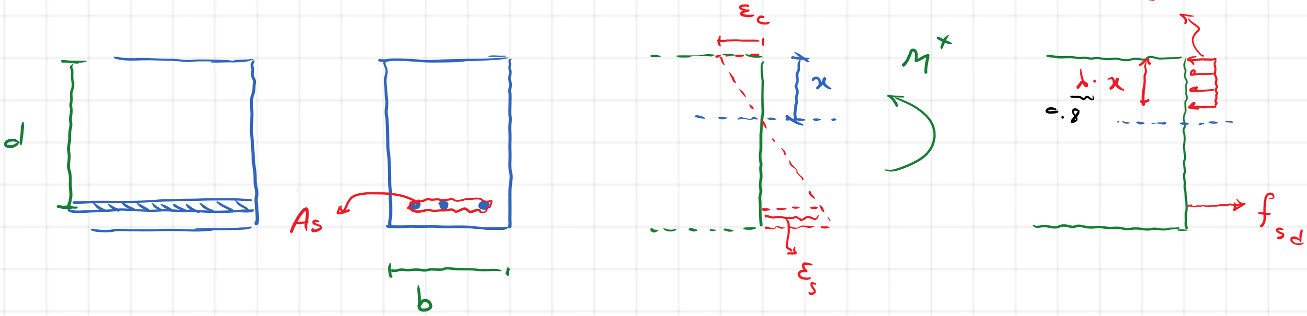
$f_{yd} = \frac{f_{yk}}{\gamma_s} = \frac{500\ MPa}{1.15} = 435\ MPa$

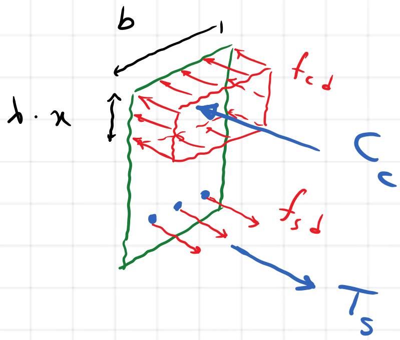
$\varepsilon_{yd} = \frac{f_{yd}}{E_s} = \frac{435\ MPa}{200\ GPa} = 0.002175$

Figure 3.8: Idealised and design stress-strain diagrams for reinforcing steel (for tension and compression)

(4) The design value of the modulus of elasticity, E_s may be assumed to be 200 GPa.

calculation of balance reinforcement: $(f_{ck} \leq 50\ \text{MPa})$

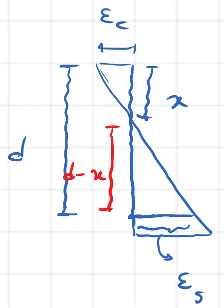




$$\begin{cases} C_c = f_{cd} \cdot b \cdot \lambda x \\ T_s = f_{sd} \cdot A_s \end{cases}$$

Equilibrium: $C_c = T_s$

$$\boxed{f_{cd} \cdot b \cdot \lambda \cdot x = f_{sd} \cdot A_s} \quad (1)$$



$$\frac{\varepsilon_c}{x} = \frac{\varepsilon_s}{d-x} \rightarrow \boxed{\frac{x}{d-x} = \frac{\varepsilon_c}{\varepsilon_s}} \quad (2)$$

$\varepsilon_c \rightarrow$ is limited to $\boxed{\varepsilon_{cu} = 0.0035}$

$$(2) \quad \frac{x}{d-x+x} = \frac{\varepsilon_c}{\varepsilon_s+\varepsilon_c} \Rightarrow x = \frac{\varepsilon_c}{\varepsilon_s+\varepsilon_c} \cdot d$$

Balance condition :

when Steel yields concrete approaches to it ultimate strain.

Steel → yield → $\begin{cases} f_{sd} = f_{yd} \\ \varepsilon_{sd} = \varepsilon_{yd} \end{cases}$

Concrete → $\varepsilon_c = \varepsilon_{cu}$

$$x = \frac{\varepsilon_c}{\varepsilon_s + \varepsilon_c} \cdot d$$

$$x_b = \frac{0.0035}{\varepsilon_{yd} + 0.0035} \cdot d$$

$\begin{cases} f_{yk} = 500 \text{ MPa} \\ f_{yd} = 435 \text{ MPa} \\ \varepsilon_{yd} = 0.002175 \end{cases}$

$$\boxed{x_b = 0.617\,d}$$

① → $f_{cd} \cdot b \cdot \underset{\downarrow\, 0.8}{\lambda} \cdot \underset{\downarrow\, x_b = 0.617d}{x} = \overset{f_{yd}}{f_{sd}} \cdot A_{s_b}$ → $A_{sb} = \dfrac{f_{cd}}{f_{yd}} \cdot b \cdot 0.8 \cdot 0.617 d$

$$A_{sb} = \underbrace{0.8 \times 0.617}_{0.493} \times \frac{f_{cd}}{f_{yd}} \cdot b \cdot d \rightarrow \overbrace{\frac{A_{sb}}{b \cdot d} \cdot \frac{f_{yd}}{f_{cd}}}^{\omega} = 0.493$$

$$\omega \rightarrow \omega_b \rightarrow \boxed{\beta_{bd} = 0.493}$$

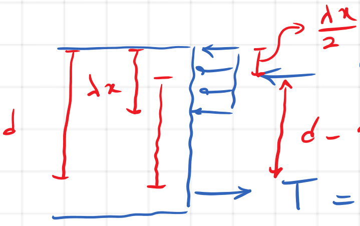
if $\omega = \dfrac{A_s}{b \cdot d} \cdot \dfrac{f_{yd}}{f_{cd}} < \beta_{bd}$ → low steel → failure will be due to steel yielding
→ ductile failure

if $\omega = \dfrac{A_s}{b \cdot d} \cdot \dfrac{f_{yd}}{f_{cd}} > \beta_{bd}$ → over steel ratio (high) → failure will be due to concrete crash → brittle failure

Shah.fi

$$\omega = \frac{A_s}{b \cdot d} \cdot \frac{f_{yd}}{f_{cd}}$$

$$\beta_{bd} = \frac{A_{sb}}{b \cdot d} \cdot \frac{f_{yd}}{f_{cd}} = 0.493$$



$$C_c = f_{cd} \cdot b \cdot \lambda \cdot x$$

$$T = f_{sd} \cdot A_s$$

$$M = (C_c \text{ or } T) \cdot \left(d - \frac{\lambda x}{2}\right)$$

$$M = f_{cd} \cdot b \cdot \lambda \cdot x \left(d - \frac{\lambda x}{2}\right) \Rightarrow$$

$$M = f_{cd} \cdot b \cdot \lambda \cdot x \cdot d\left(1 - \frac{\lambda x}{2d}\right)$$

$$C_c = T \rightarrow f_{cd} \cdot b \cdot \lambda \cdot x = f_{sd} \cdot A_s$$

$$\lambda \cdot x = \underbrace{\frac{f_{sd}}{f_{cd}} \cdot \frac{A_s}{b\, d}}_{\beta} \cdot d \rightarrow \boxed{\lambda x = \beta \cdot d}$$

$$M = f_{cd} \cdot b \cdot \beta \cdot d \cdot d \left(1 - \frac{1}{2} \cdot \beta\right)$$

$$M = f_{cd} \cdot b \cdot d^2 \cdot \underbrace{\beta\left(1 - \frac{1}{2}\beta\right)}_{\mu}$$

$$M = f_{cd} \cdot b \cdot d^2 \cdot \mu, \quad \mu = \beta\left(1 - \frac{1}{2}\beta\right), \quad \beta = \frac{f_{sd}}{f_{cd}} \cdot \frac{A_s}{b \cdot d}$$

f_{yd}

$\beta_{bd} = 0.493$

$\beta = \frac{A_s}{b \cdot d} \cdot \frac{f_{yd}}{f_{cd}}$

$\mu = \beta \left(1 - \frac{\beta}{2}\right)$

$M = \mu \cdot f_{cd} \cdot b \cdot d^2$

if $\left.\begin{matrix} A_s > A_{sb} \\ (\beta > \beta_{bd}) \end{matrix}\right\} \rightarrow \beta = \min \left\{ \underbrace{\beta}_{\frac{A_s}{b \cdot d} \cdot \frac{f_{yd}}{f_{cd}}}, \beta_{bd} \right\}$