For a given rectangular concrete beam, according to Eurocode 1992-1-1:

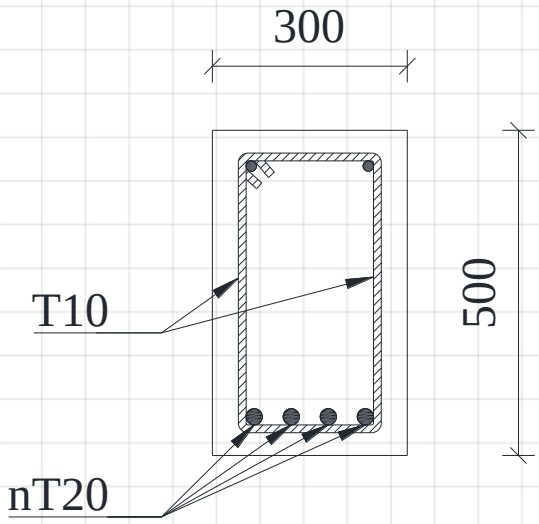
a) Derive the procedure of determining the required reinforcement.

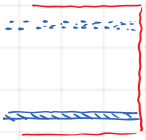
A concrete rectangular cross-section, 300x500, is made of C30/37 and reinforced by B500 steel. Assume the stirrup is T10 and the required nominal cover is 35mm:

b) If the available reinforcement is T20, how many reinforcements are required to withstand $250kN.m$ bending moment?
c) Determine the stress in reinforcements based on your design.

PS. Neglect the effect of compressive steel.

Dimensions are in mm. The cross-section is under a positive bending moment. The sketch is schematic and does not necessarily means that 4T20 is sufficient.





M_{Ed}

$$M_{Rd} = \mu \cdot f_{cd} \cdot b \cdot d^2$$

$$\mu = \beta \left(1 - \frac{\beta}{2}\right)$$

$$\omega = \frac{A_s}{b \cdot d} \cdot \frac{f_{yd}}{f_{cd}}, \quad \beta_{bd} = 0.493 \quad (f_{yk} = 500\ MPa)$$

$$M_{Rd,c} = \mu_{bd} \cdot f_{cd} \cdot b \cdot d^2$$

$$\mu_{bd} = \beta_{bd}\left(1 - \frac{\beta_{bd}}{2}\right) = 0.493\left(1 - \frac{0.493}{2}\right) = 0.372$$

$M_{Ed} = 250\ kN.m < M_{Rd,c} = 300\ kN.m \rightarrow$ Tensile reinforcement would be enough.

$M_{Ed} = 400\ kN.m > M_{Rd,c} = 300\ kN.m \rightarrow$

- using better concrete class
- increasing beam dimensions → $\begin{cases} b \\ h \end{cases} \rightarrow$
- we can check if by reasonable compressive reinforcement the beam can take M_{Ed}

① Determine the $M_{Rd,c}$ (Maximum bending moment capacity of concrete beam without compressive reinforcement)

② If $M_{Ed} < M_{Rd,c} \rightarrow$ Compressive reinforcement is not needed.

③ $M_{Ed} = \mu \cdot f_{cd} \cdot b \cdot d^2 \rightarrow \mu = \dfrac{M_{Ed}}{f_{cd} \cdot b \cdot d^2}$, $(\mu < \mu_{bd})$

$\mu = \beta\left(1 - \frac{\beta}{2}\right) \Rightarrow \beta^2 - 2\beta + 2\mu = 0$

$\mu = \beta - \frac{\beta^2}{2}$

$2\mu = 2\beta - \beta^2$

$$\beta = \frac{2 \pm \sqrt{4 - 8\mu}}{2} = \frac{2 \pm 2\sqrt{1 - 2\mu}}{2} = 1 \overset{\times}{\pm} \sqrt{1 - 2\mu} \Rightarrow$$

④ $\beta = 1 - \sqrt{1 - 2\mu}$

$$\beta = \frac{A_s}{b \cdot d} \cdot \frac{f_{yd}}{f_{cd}} = \frac{\frac{T}{\varepsilon_c}}{d \cdot \frac{\varepsilon_c}{\lambda x}} = \frac{\lambda x}{d}$$

$C_c = f_{cd} \cdot b \cdot \lambda x \rightarrow f_{cd} \cdot b = \dfrac{C_c}{\lambda x}$

⑤ $\beta = \frac{A_s}{b.d} \cdot \frac{f_{yd}}{f_{cd}} \Rightarrow A_s = \dots \longrightarrow A_{s.required}$

$A_{s\,min} = \max\left\{0.26\,\frac{f_{ctm}}{f_{yd}}\,;\,0.0013\right\} \cdot b.d$

Strength classes for concrete															Analytical relation / Explanation
f_{ck} (MPa)	12	16	20	25	30	35	40	45	50	55	60	70	80	90	
$f_{ck,cube}$ (MPa)	15	20	25	30	37	45	50	55	60	67	75	85	95	105	
f_{cm} (MPa)	20	24	28	33	38	43	48	53	58	63	68	78	88	98	$f_{cm} = f_{ck}+8$(MPa)
f_{ctm} (MPa)	1,6	1,9	2,2	2,6	2,9	3,2	3,5	3,8	4,1	4,2	4,4	4,6	4,8	5,0	$f_{ctm}=0,30 \cdot f_{ck}^{(2/3)} \leq C50/60$ $f_{ctm}=2,12 \cdot \ln(1+(f_{cm}/10)) > C50/60$

$A_s = \max\{A_{s.required}\,,\,A_{s.min}\}$

$\phi_s = ?\,,\quad n = \dots$

Width of the beam	$b := 300\ mm$
Height of the beam	$h := 500\ mm$
Nominal cover	$c_{nom} := 35\ mm$
Reinforcement Diameter	$\phi_s := 20\ mm$
Stirrup Diameter	$\phi_{sw} := 10\ mm$
Design Bending Moment	$M_{Ed} := 250\ kN \cdot m$
Concrete Compressive Strength	$f_{ck} := 30\ MPa$

$A_{s.rqd} = 1520\ mm^2$

$A_{s.provided} = 5\,T20 = \frac{5 \times \pi \times (20.mm)^2}{4}$

$A_{s.pr} = 1571\ mm^2$

$f_{sd} = \frac{1520\ mm^2}{1571\ mm^2} \cdot f_{yd}\ (435\ MPa) = 421\ MPa$

$Status_1$ = "Compressive Reinforcement is not Required"

$A_s = 1520\ mm^2$

Required Reinforcement (Number) $n_{s.rqd} = 4.84$

$n := 5$