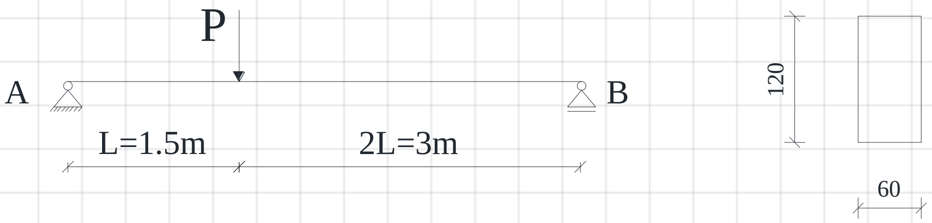
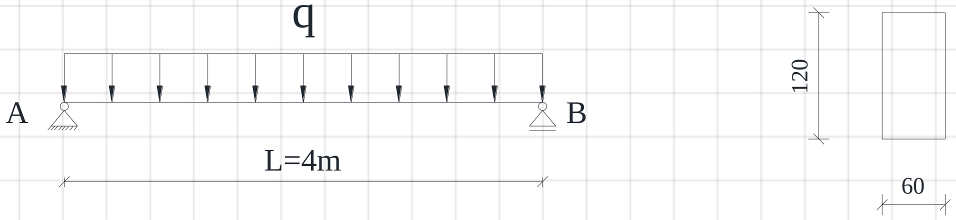
In this video, the definition of a determinate beam to become a mechanism is introduced. Three beams, two under point load and one under distributed load, will be solved to determine the elastic and plastic loads. Moreover, the length of the beam before becoming a mechanism will also be studied to determine the elastic and partially plastic length of the beam. In the first two cases, the rectangular cross-section with a width of 60mm and height of 120mm made of an elastic perfectly plastic material with $f_y = 210MPa$ is assumed. For the third one, use only the M_{el} and M_{pl} parametrically. Dimensions are in mm unless otherwise specified.

For parts a) and b), determine the elastic and plastic load and then determine the length of the beam, which will be elastic when the load approaches the plastic load. In part c) determine the elastic and plastic load and determine which point will be plastic first.

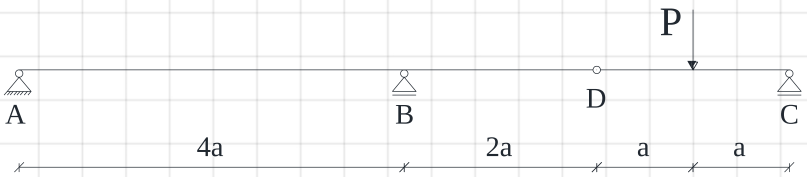
a) A simple beam with the force P at its one-third distance from the left support.

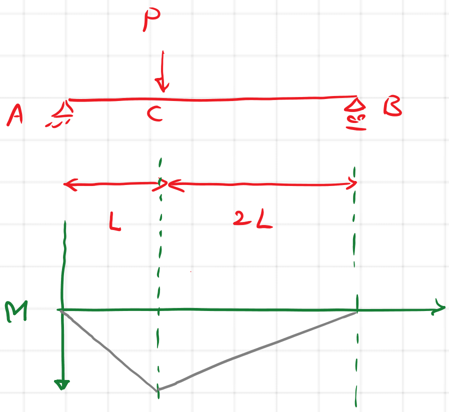


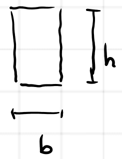
b) A simple beam under distributed load q.



c) A two spans beam with a mechanical hinge in the middle of the right's span, as shown below.







$W_{pl} = \frac{bh^2}{4}$

$M_{pl} = W_{pl} \cdot f_y$

$b = 60\ mm$
$h = 120\ mm$
$f_y = 210\ MPa$

$M_{Max} = \frac{P \cdot L \cdot (2L)}{3L} = \frac{2PL}{3}$

$W_{pl} = 216000\ mm^3$

$M_{pl} = W_{pl} \cdot f_y = 45.36\ kN \cdot m$

$W_{el} = \frac{bh^2}{6} = 144000\ mm^3$

$M_{el} = W_{el} \cdot f_y = 30.24\ kN \cdot m$

$(L = 1.5m)$

① $\frac{2PL}{3} = 30.24\ kN \cdot m \rightarrow P_{el} = 30.24\ kN$ (L → 1.5m)

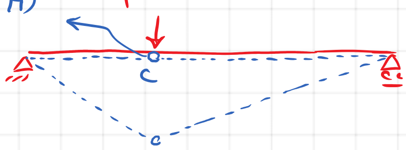
② $\frac{2P \cdot L}{3} = 45.36\ kN \cdot m \rightarrow P_{pl} = 45.36\ kN$

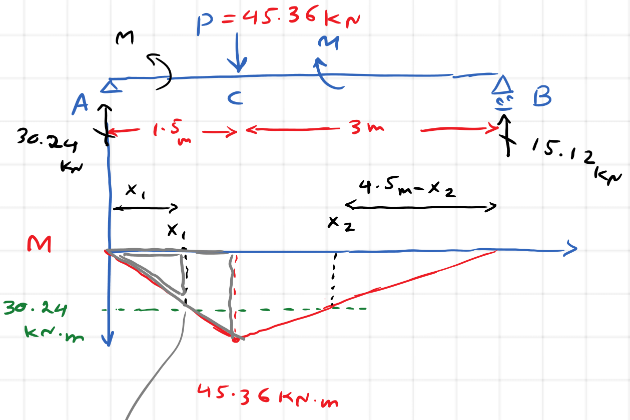
$P < 30.24\ kN \rightarrow$ Elastic behavior

$30.24\ kN < P < 45.36\ kN \rightarrow$ partially plastic

$P > 45.36\ kN \rightarrow$ at point c → no rotational resistance
→ hinge behavior
→ (plastic hinge)

(P.H) $P = 45.36\ kN$





$M_{el} = 30.24\ kN \cdot m$

$M = 30.24 \cdot x_1 = 30.24\ kN \cdot m$

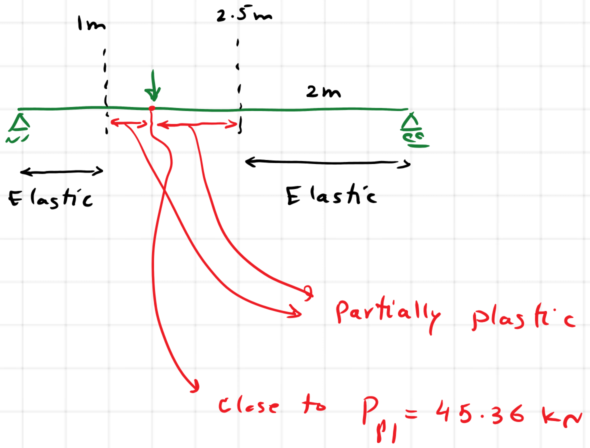
$\rightarrow \boxed{x_1 = 1m}$

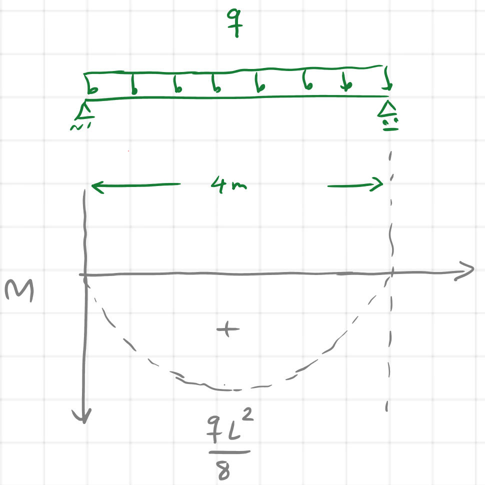
$M = 15.12 \cdot (4.5 - x_2) = 30.24$

$x_2 = 2.5m$

$$\frac{45.36\ kN \cdot m}{1.5\ m} = \frac{30.24\ kN \cdot m}{x_1}$$

$\boxed{x_1 = 1m}$





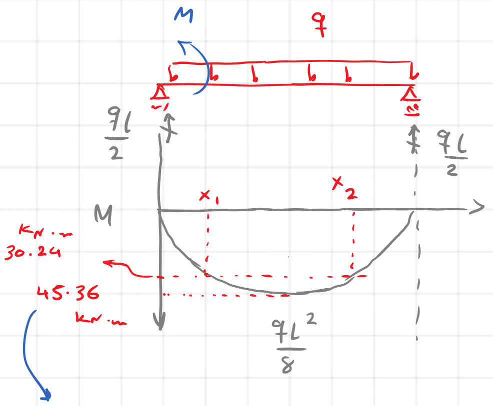
$M_{el} = 30.24\ kN \cdot m$

$M_{pl} = 45.36\ kN \cdot m$

$(L = 4m)$

$\frac{qL^2}{8} = 30.24\ kN \cdot m \rightarrow q_{el} = 15.12\ kN/m$

$\frac{qL^2}{8} = 45.36\ kN \cdot m \rightarrow q_{pl} = 22.68\ kN/m$



$q_{P1} = 22.68$ kN/m

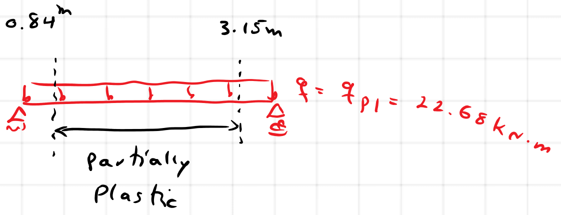
$$M = \frac{ql}{2} \cdot x - \frac{q \cdot x^2}{2}$$

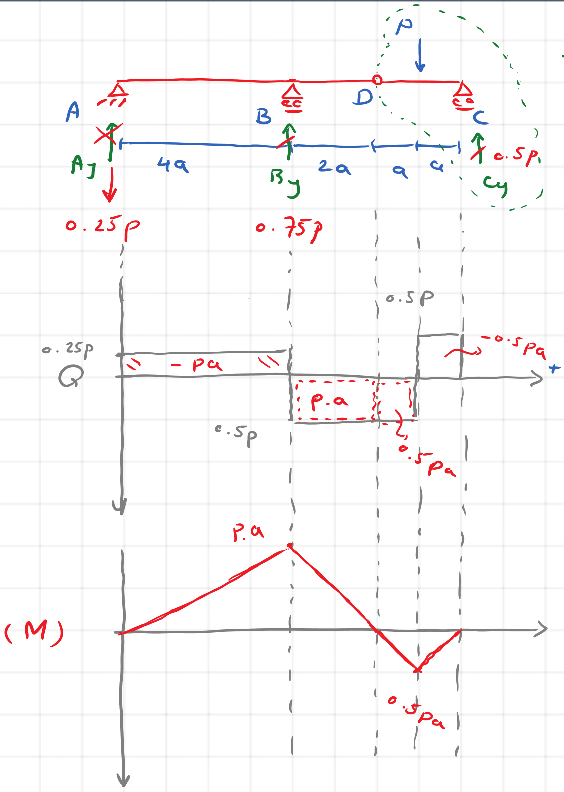
$$30.24 = 22.68 \left(2 \cdot x - \frac{x^2}{2} \right)$$

$$\rightarrow \left(\frac{4}{3} = 2x - \frac{x^2}{2} \right) \cdot 2$$

$$x^2 - 4x + \frac{8}{3} = 0$$

$$x = \frac{4 \pm \sqrt{16 - 4 \cdot 8/3}}{2} = \frac{4 \pm 4\sqrt{1 - 2/3}}{2} = 2 \pm 2\sqrt{1/3} \begin{cases} 3.15 \text{ m} \\ 0.84 \text{ m} \end{cases}$$





$$\circlearrowleft + \sum M_D = 0 \quad (\text{Right}) \qquad -P\cdot a + C_y \cdot 2a = 0 \Rightarrow C_y = P/2$$

$$\circlearrowleft + \sum M_A = 0 \qquad B_y \cdot 4a - P\cdot 7a + C_y \cdot 8a = 0 \quad (C_y = P/2)$$

$$B_y = \frac{3}{4} P = 0.75 P$$

$$+\uparrow \sum F_y = 0 \qquad A_y + B_y + C_y - P = 0 \quad (B_y = 0.75P,\ C_y = P/2)$$

$$\boxed{A_y = -0.25 P}$$

$$M_{max} = P\cdot a = M_{el} \rightarrow \boxed{P_{el} = \frac{M_{el}}{a}}$$

$$= M_{pl} \rightarrow \boxed{P_{pl} = \frac{M_{pl}}{a}}$$