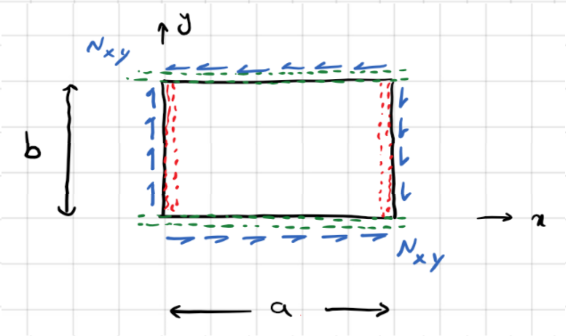
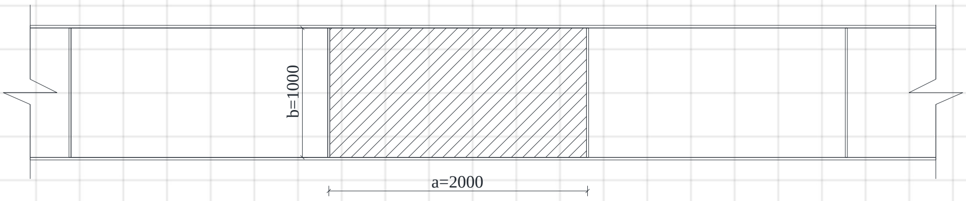
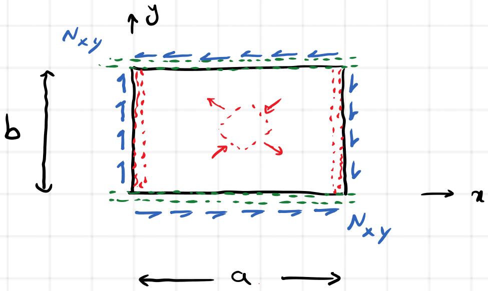
A rectangular plate with the dimensions of a and b is subjected to pure shear force, as shown in the figure. Assume the deformation shape function in shear buckling is:



$$w = w_0 . \sin\left(\frac{\pi x}{a}\right) . \sin\left(\frac{\pi y}{b}\right) . \sin\left(\frac{\pi x}{a} - \frac{\pi y}{b}\right)$$

a) Derive the total potential energy
b) Determine the integration domain.
c) Solve the total potential energy and find the buckling load
d) If the plate is with a thickness of 6mm, $a = 2m$, and $b = 1m$, assuming $E = 200GPa$, $v = 0.3$ determine the critical buckling load and compare the results with the given equations in codes.



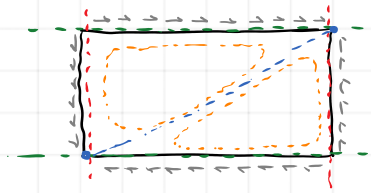


$$w(x,y) = w_0 \sin\left(\frac{\pi x}{a}\right) \cdot \sin\left(\frac{\pi \cdot y}{b}\right) \cdot \sin\left(\frac{\pi x}{a} - \frac{\pi y}{b}\right)$$

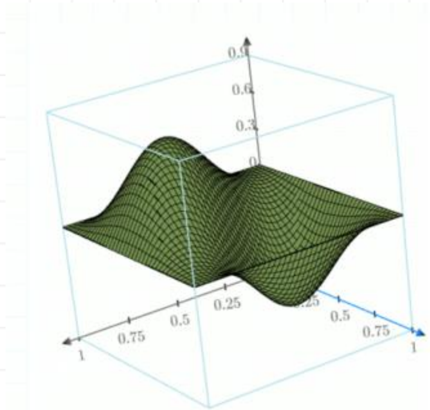
$\sin\left(\frac{\pi x}{a}\right) = 0 \rightarrow x = 0 \,,\; x = a$

$\sin\left(\frac{\pi y}{b}\right) = 0 \rightarrow y = 0 \,,\; y = b$

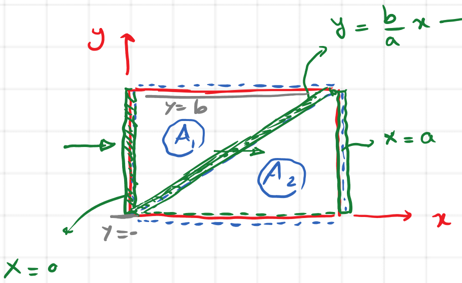
$\sin\left(\frac{\pi x}{a} - \frac{\pi y}{b}\right) = 0 \rightarrow b \cdot x = a \cdot y \rightarrow y = \frac{b}{a} \cdot x$



$$U = \frac{1}{2} D \int_A w_{xx}^2 + w_{yy}^2 + 2\nu \cdot w_{xx} \cdot w_{yy} + 2(1-\nu) \cdot w_{xy}^2 \; dA$$



$$V = \frac{1}{2} \int \cancel{N_x \cdot w_x^2} + \cancel{N_y \, w_y^2} + 2 N_{xy} \cdot w_x \, w_y$$



$$\int_A F \underbrace{dA}_{dx\,dy} = \int_{A_1} F \cdot dA_1 + \int_{A_2} F \cdot dA_2$$

$$\int_{y=0}^{y=b} \int_{x=0}^{x=\frac{a}{b} \cdot y} F \; dx \; dy + \int_{y=0}^{y=b} \int_{x=\frac{a}{b} \cdot y}^{x=a} F \cdot dx \cdot dy$$

$$N_{xy.cr}(a,b,v,D) := \frac{\partial^2}{\partial {w_0}^2} \Pi(w_0,a,b,v,D,N_{xy}) = 0 \xrightarrow{solve, N_{xy}} \frac{(4 \cdot D \cdot b^4 + 4 \cdot D \cdot a^2 \cdot b^2 + 4 \cdot D \cdot a^4) \cdot \pi^2}{a^3 \cdot b^3}$$

$E := 200\ GPa$ $\quad t := 6\ mm$ $\quad b := 1000\ mm$ $\quad a := 2000\ mm$ $\quad v := 0.3$

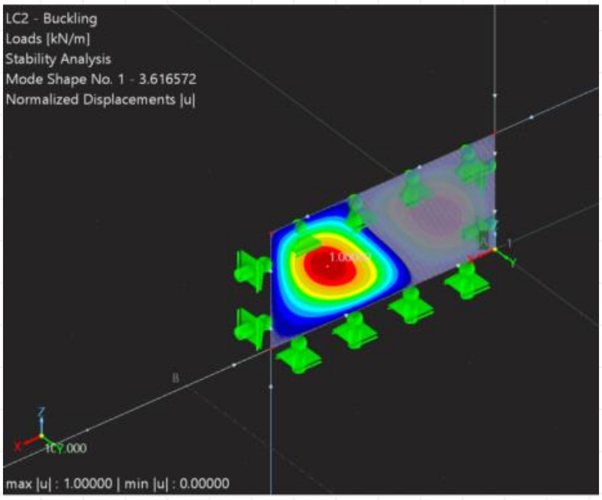
$$D := \frac{E \cdot t^3}{12 \cdot (1 - v^2)} = 3.956\ kN \cdot m$$

$$N_{xy.cr}(a,b,v,D) = 409.968\ \frac{kN}{m}$$

$$k_\tau := 5.35 + \frac{4}{\left(\frac{a}{b}\right)^2} = 6.35 \qquad N_{cr} := k_\tau \cdot \frac{\pi^2 \cdot D}{b^2} = 247.933\ \frac{kN}{m}$$

$$N_{xy,cr} = \frac{4\pi^2 D (a^4 + a^2 b^2 + b^4)}{a^3 b^3}$$

$$\left\{ \begin{array}{l} a = 2m \\ b = 1m \\ t = 6\ mm \\ E = 200\ GPa \end{array} \right. \rightarrow N_{xy,cr} = 410\ kN/m$$



RFEM: $\quad$ L.F = 3.62

Applied load: 100 kN/m

$$\rightarrow \boxed{N_{cr} = 362\ kN/m}$$

$N_{xy,cr} = 410\ kN/m$, $N_{cr,RFEM} = 362\ kN/m$ $\rightarrow$ error: $\frac{410 - 362}{362} = \frac{48}{362} = 13.2\%$

$N_{xy.cr} = 410\ kN/m$, $N_{cr,Code} = 250\ kN/m$ $\rightarrow$ error: $\frac{410 - 250}{250} = \frac{160}{250} = 64\%$

$N_{cr,code} = 250$, $N_{cr,RFEM} = 362\ kN/m$ $\rightarrow$ error: $\frac{362 - 250}{362} = \frac{112}{362} = 31\%$