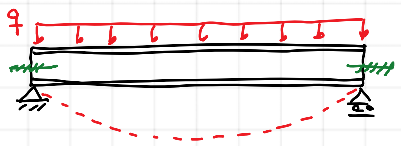
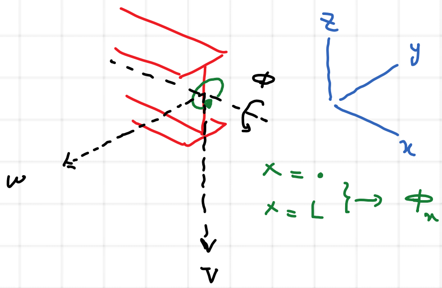
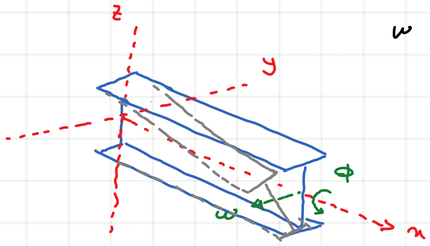
This series of videos teaches the calculation of buckling load for a simply supported beam under a distributed load. As you may noticed, Eurocode does not provide an equation for critical bending moment in lateral torsional buckling calculation.

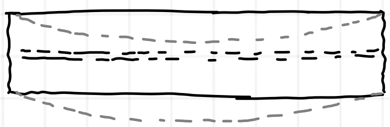
This video presents a calculation of buckling load using total potential energy. Subsequent videos will cover buckling load calculations using Ansys and RFEM.



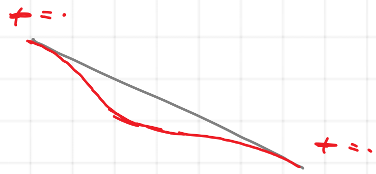


$x=0$
$x=L$ $\} \rightarrow \varphi_x = 0$





w → half Sin



φ → Cos

$$w(x) = A \sin\left(\frac{\pi x}{L}\right) \rightarrow$$

$x = 0 \rightarrow w(0) = 0$

$x = L \rightarrow w(L) = 0$

$x = \frac{L}{2} \rightarrow w\left(\frac{L}{2}\right) = A$

$$\varphi(x) = B \cdot \left[\cos\left(\frac{2\pi x}{L}\right) - 1\right] \rightarrow$$

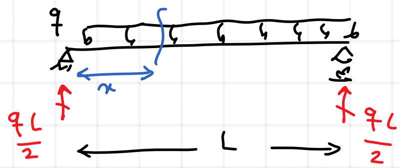
$x = 0 \rightarrow \varphi(0) = 0$

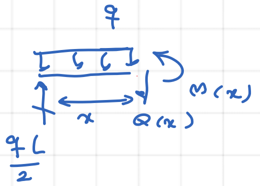
$x = L \rightarrow \varphi(L) = 0$

$x = \frac{L}{2} \rightarrow \varphi\left(\frac{L}{2}\right) = -B$

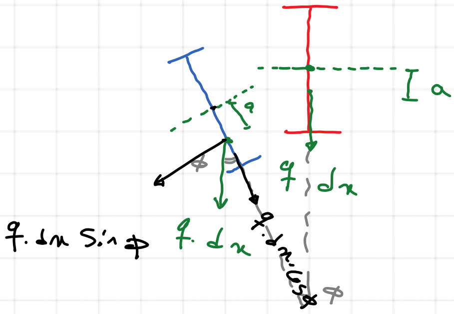
$$\pi = \frac{1}{2}\int_0^L E\, I_z \cdot w''(x)^2\, dx + \frac{1}{2}\int_0^L G\, I_t \cdot \varphi'(x)^2\, dx$$

$$+ \frac{1}{2}\int_0^L E\, I_\omega\, \varphi''(x)^2\, dx - \int_0^L \left(M_y(x)\, \varphi(x)\right)'\, w'(x)\, dx + \sum_0^L \frac{1}{2}\, T(x)\, \varphi(x)$$



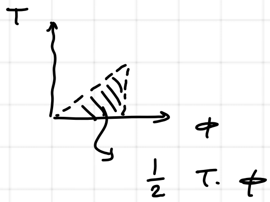


$$\boxed{M_y(x) = \frac{qL}{2}\cdot x - \frac{q\cdot x^2}{2}}$$



$T(x) = q\cdot dx \cdot \sin\phi \cdot a$

ϕ



$$\Pi = \frac{1}{2}\int_0^L E I_z \cdot w''(x)^2\, dx + \frac{1}{2}\int_0^L G I_t \cdot \phi'(x)^2\, dx$$

$$+ \frac{1}{2}\int_0^L E I_\omega \phi''(x)^2\, dx - \int_0^L \left(M_y(x)\, \phi(x)\right)' w'(x)\, dx + \int \frac{1}{2} T(x)\, \phi(x)\, dx$$

$w(x) = A \sin\left(\frac{\pi \cdot x}{L}\right)$

$\phi(x) = B\left[\cos\left(\frac{2\pi x}{L}\right) - 1\right]$

$M_y(x) = \frac{qL}{2} x - \frac{q x^2}{2}$

$T(x) = q\cdot \phi(x) \cdot a$

$$w(x,l,A) := A \cdot \sin\left(\frac{\pi \cdot x}{l}\right)$$

$$\phi(x,l,B) := B \cdot \left(\cos\left(\frac{2 \cdot \pi \cdot x}{l}\right) - 1\right)$$

$$M_y(x,l,q) := \frac{q \cdot l}{2} \cdot x - \frac{q \cdot x^2}{2}$$

$$T(x,l,B,q,a) := q \cdot \phi(x,l,B) \cdot a$$

$$\Pi(A,B,l,E,G,I_z,I_t,I_w,q,a) := \frac{1}{2} \cdot \int_0^l E \cdot I_z \cdot \left(\frac{\partial^2}{\partial x^2} w(x,l,A)\right)^2 \mathrm{d}x + \frac{1}{2} \cdot \int_0^l G \cdot I_t \cdot \left(\frac{\partial}{\partial x} \phi(x,l,B)\right)^2 \mathrm{d}x$$
$$+ \frac{1}{2} \cdot \int_0^l E \cdot I_w \cdot \left(\frac{\partial^2}{\partial x^2} \phi(x,l,B)\right)^2 \mathrm{d}x - \int_0^l \left(\frac{\partial}{\partial x}\left(M_y(x,l,q) \cdot \phi(x,l,B)\right)\right) \cdot \left(\frac{\partial}{\partial x} w(x,l,A)\right) \mathrm{d}x$$
$$+ \frac{1}{2} \cdot \int_0^l T(x,l,B,q,a) \cdot \phi(x,l,B) \, \mathrm{d}x$$

$$\frac{\partial}{\partial A} \Pi(A,B,l,E,G,I_z,I_t,I_w,q,a) \rightarrow \frac{I_z \cdot A \cdot E \cdot \pi^4}{2 \cdot l^3} + \frac{80 \cdot B \cdot l \cdot q}{27 \cdot \pi}$$

$$\frac{\partial}{\partial B} \Pi(A,B,l,E,G,I_z,I_t,I_w,q,a) \rightarrow \frac{8 \cdot I_w \cdot B \cdot E \cdot \pi^4}{l^3} + \frac{2 \cdot I_t \cdot B \cdot G \cdot \pi^2}{l} + \frac{3 \cdot B \cdot a \cdot l \cdot q}{2} + \frac{80 \cdot A \cdot l \cdot q}{27 \cdot \pi}$$

$$Mat(l,E,G,I_z,I_t,I_w,q,a) := \begin{bmatrix} \dfrac{I_z \cdot E \cdot \pi^4}{2 \cdot l^3} & \dfrac{80 \cdot l \cdot q}{27 \cdot \pi} \\ \dfrac{80 \cdot l \cdot q}{27 \cdot \pi} & \dfrac{8 \cdot I_w \cdot E \cdot \pi^4}{l^3} + \dfrac{2 \cdot I_t \cdot G \cdot \pi^2}{l} + \dfrac{3 \cdot a \cdot l \cdot q}{2} \end{bmatrix}$$

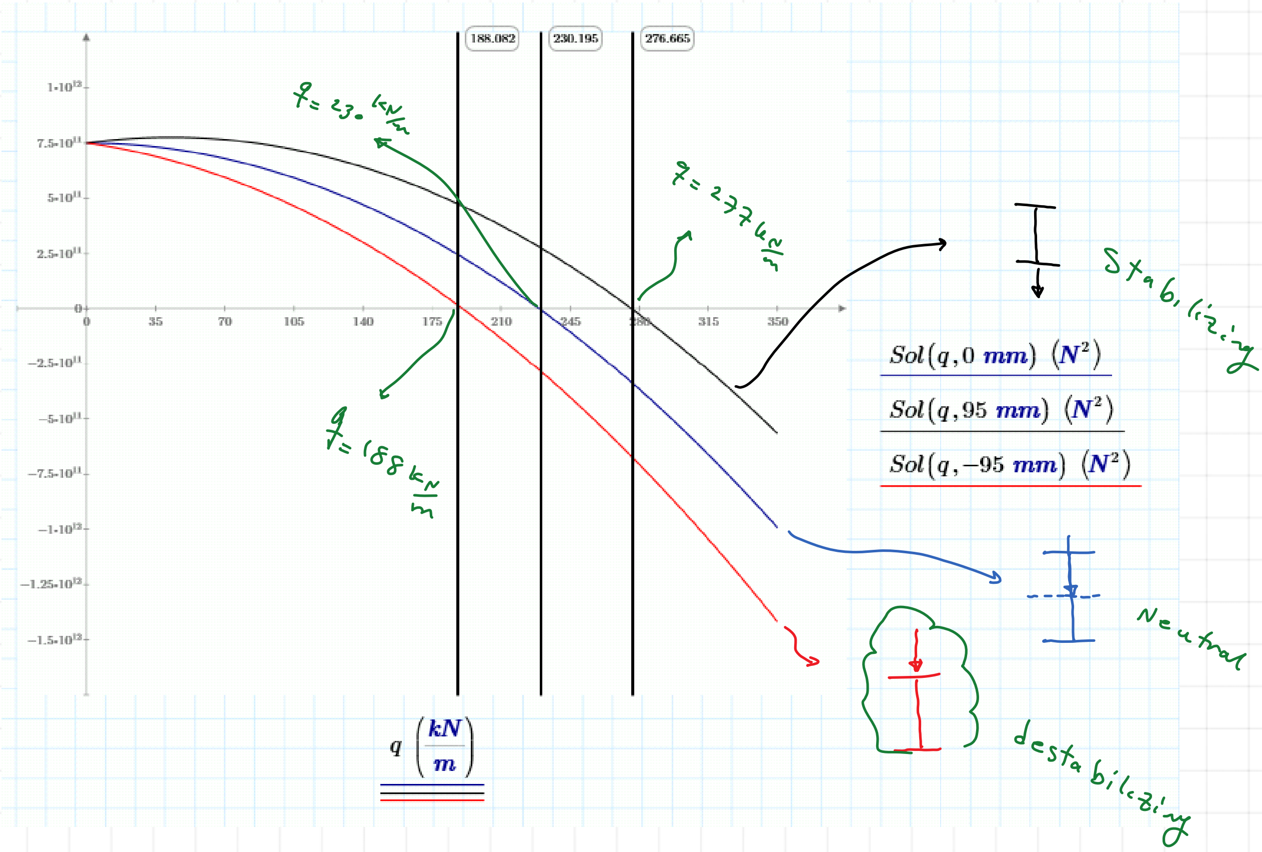
Profile	[Sh]ear [are]a z-z $A_{v,z}$ [m]m²] (η=1.2)	Shear area y-y $A_{v,y}$ [mm²]	Second moment of area I_y [×10⁶ mm⁴]	Radius of gyration i_y [mm]	Elastic section modulus $W_{el,y}$ [×10³ mm³]	Plastic section modulus $W_{pl,y}$ [×10³ mm³]	Second moment of area I_z [×10⁶ mm⁴]	Radius of gyration i_z [mm]	Elastic section modulus $W_{el,z}$ [×10³ mm³]	Plastic section modulus $W_{pl,z}$ [×10³ mm³]	Torsion constant I_T [×10³ mm⁴]	Torsion modulus W_T [×10³ mm³]	Warping constant I_w [×10⁶ mm⁶]	Warping modulus W_w [×10³ mm⁴]
HEA200	808	4000	36.92	82.8	388.6	429.5	13.36	49.8	133.6	203.8	204.3	31.43	105580	11830

$I_z := 13.36 \cdot 10^6 \cdot mm^4$ $\quad I_t := 204.3 \cdot 10^3 \cdot mm^4$ $\quad I_w := 105580 \cdot 10^6 \cdot mm^6$

$E := 210\ GPa$ $\quad v := 0.3$ $\quad G := \dfrac{E}{2\,(1+v)}$ $\quad l := 4\ m$

$$Sol(q,a) := \det\left(Mat(l,E,G,I_z,I_t,I_w,q,a)\right)$$

$$q := 0, 0.1 \frac{kN}{m} .. 350 \frac{kN}{m}$$



Des: → $M_{cr} = \frac{qL^2}{8} = \frac{188\,kN/m \times (4m)^2}{8} = 376\ kN\cdot m$

Neu: → $M_{cr} = \frac{230\,kN/m \times (4m)^2}{8} = 460\ kN\ m$

Sta. → $M_{cr} = 277\,kN/m \times \frac{(4m)^2}{8} = 554\ kN\ m$