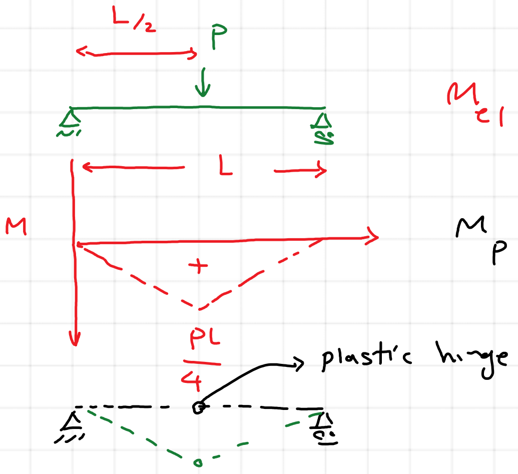
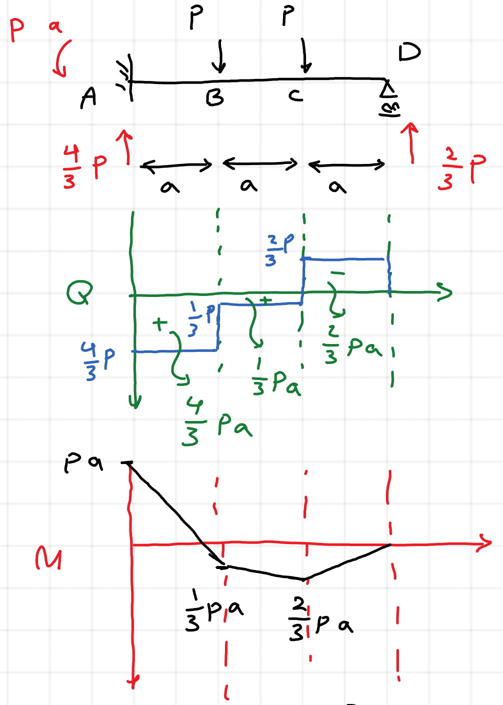
In this series of videos, we aim to teach how to calculate the beam collapse load. As you may know, if a plastic hinge is formed in an indeterminate beam, considering the indeterminacy degree, the beam could collapse or even take more loads before the structure mechanism occurs. This video teaches you the calculation of collapse load for indeterminate beams under point load.

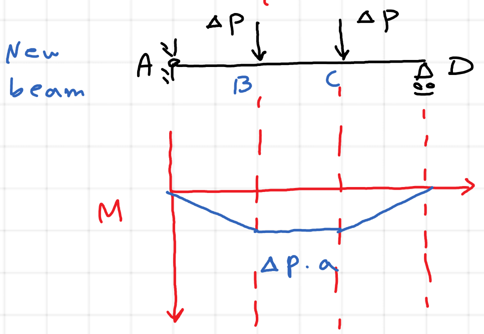


$$M_{el} = \sigma_y \, W_{el}$$

$$M_{pl} = \sigma_y \, W_{pl}$$



$Max(M) = P\,a = M_p \rightarrow P_1 = \frac{M_p}{a} \rightarrow$ point A becomes plastic hinge



A : $P\,a + 0 = M_p$ (1)

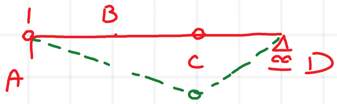
B : $\frac{1}{3} P\,a + \Delta P\,a$

C : $\frac{2}{3} \underbrace{P \cdot a}_{M_p} + \Delta P\,a = M_p \rightarrow \Delta P \cdot a = \frac{1}{3} M_p$

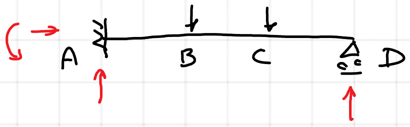
D : 0 (1)

$$\Delta P = \frac{1}{3} \frac{M_p}{a}$$

$$\rightarrow P_{max} = P_1 + \Delta P = \frac{M_p}{a} + \frac{1}{3} \frac{M_p}{a} = \frac{4}{3} \frac{M_p}{a}$$



2 plastic hinges can form



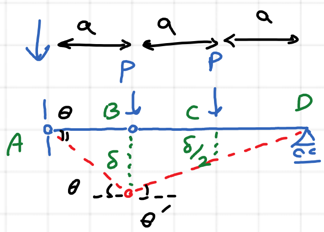
Un. 4

$$Eq.\begin{Bmatrix}\Sigma F_x = \\ \Sigma F_y = \\ \Sigma M = \end{Bmatrix} \rightarrow 3$$

Indeterminate D. $4-3=1$

Number of plastic hinges: $1+1=2$

A, B   A, C   B, C



$$EW = \Sigma P\cdot\delta = P\cdot\delta + P\cdot\frac{\delta}{2} = \frac{3}{2}P\cdot\delta$$

$$IW = \Sigma M_p\,\theta = \underbrace{M_p\,\theta}_{A} + \underbrace{M_p\,\theta}_{B\,\text{Left}} + \underbrace{M_p\,\theta'}_{B\,\text{Right}}$$

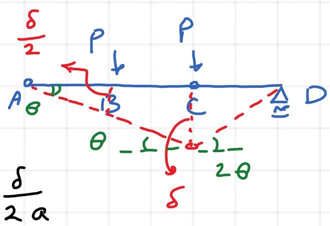
$$\theta = \frac{\delta}{a} \qquad \theta' = \frac{\delta}{2a}$$

$$\theta a = \theta' 2a$$

$$\boxed{\theta' = \theta/2}$$

$$IW = M_p(\theta + \theta + \theta/2) = \frac{5}{2}M_p\,\theta$$

$$EW = IW \rightarrow \frac{3}{2}P\,\delta = \frac{5}{2}M_p\,\theta \quad \left(\theta = \frac{\delta}{a}\right) \rightarrow \boxed{P = \frac{5}{3}\frac{M_p}{a}}$$



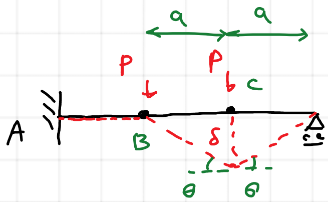
$$\theta = \frac{\delta}{2a}$$

$$EW = \Sigma P\cdot\delta = P\frac{\delta}{2} + P\,\delta = \frac{3}{2}P\cdot\delta$$

$$IW = \Sigma M_p\,\theta = \underbrace{M_p\,\theta}_{A} + \underbrace{M_p\,\theta}_{B\,\text{left}} + \underbrace{M_p\,2\theta}_{B\,\text{Right}} = 4M_p\,\theta$$

$$EW = IW \rightarrow \frac{3}{2}P\cdot\delta = 4M_p\,\theta \quad \left(\theta = \frac{\delta}{2a}\right) \rightarrow \boxed{P = \frac{4}{3}\frac{M_p}{a}}$$

(BC)



$EW = \sum P.\delta = P \cdot 0 + P.\delta = P.\delta$

$IW = \underbrace{M_p \cdot 0}_{B^{left}} + \underbrace{M_p\, \theta}_{B^{right}} + \underbrace{M_p\, \theta}_{C^{left}} + \underbrace{M_p\, \theta}_{C^{right}} = 3 M_p\, \theta$

$\theta = \frac{\delta}{a}$

$EW = IW \Rightarrow \quad P.\delta = 3M_p\, \theta \quad (\theta \to \frac{\delta}{a}) \quad \rightarrow \boxed{P = 3\frac{M_p}{a}}$

$P_{ult} = \min\left\{ \underbrace{P_{AB}}_{\frac{5}{3}\frac{M_p}{a}},\ \underbrace{P_{AC}}_{\frac{4}{3}\frac{M_p}{a}},\ \underbrace{P_{BC}}_{3\frac{M_p}{a}} \right\} = \frac{4}{3}\frac{M_p}{a}$