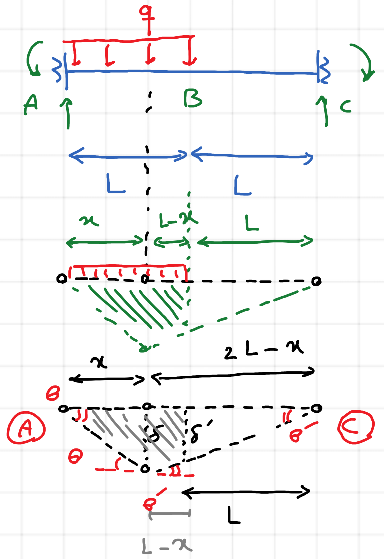
In this series of videos, we aim to teach how to calculate the beam collapse load. In the previous video, we went through the calculation of collapse load for indeterminate beams under point load. This video teaches you the calculation of collapse load for indeterminate beams under distributed load.




$\Sigma$ un : 4

$$Eq: \begin{cases} \Sigma F_y = \\ \Sigma M = \cdot \end{cases} 2 \Bigg\} \rightarrow$$ indeterminate with the degree of $4-2=2$

N of plastic h: $2+1=3$

$E_W = \Sigma p \cdot \delta = \int q\,\delta = q \cdot A$ of the load where is applied

$$\frac{\delta}{x} = \theta \qquad \frac{\delta}{2L-x} = \theta' \rightarrow \quad \theta \cdot x = \theta' \cdot (2L-x) \rightarrow \boxed{\theta' = \frac{x}{2L-x}\,\theta}$$

$$\frac{\delta}{2L-x} = \frac{\delta'}{L} \rightarrow \delta' = \delta\,\frac{L}{2L-x} \quad \text{I}$$

$$A = \frac{\delta \cdot x}{2} + \frac{\delta+\delta'}{2}\cdot(L-x) = \frac{1}{2}\cdot\left(\delta \cdot x + \left(\delta + \delta \cdot \frac{L}{2L-x}\right)(L-x)\right)$$

$$A = \frac{\delta}{2}\left(x + \left(1+\frac{L}{2L-x}\right)\cdot(L-x)\right) = \frac{\delta}{2}\left[x + \frac{2L-x+L}{2L-x}(L-x)\right]$$

$$A = \frac{\delta}{2}\left[\frac{x(2L-x)+(3L-x)(L-x)}{2L-x}\right] = \frac{\delta}{2}\left[\frac{2Lx-\cancel{x^2}+3L^2-3Lx-Lx+\cancel{x^2}}{2L-x}\right]$$

$$A = \frac{\delta}{2}\left(\frac{3L^2-2Lx}{2L-x}\right)$$

$$\boxed{E_W = q \cdot A = \frac{q\,\delta}{2}\left(\frac{3L^2-2Lx}{2L-x}\right)}$$

$$I_W = \underbrace{M_p\,\theta}_{A} + \underbrace{M_p\,(\theta+\theta')}_{B} + \underbrace{M_p\,\theta'}_{C} = M_p\left(2\theta+2\theta'\right) = 2M_p\left(\theta + \frac{x}{2L-x}\cdot\theta\right)$$

$$I_W = 2M_p\,\theta\left(1+\frac{x}{2L-x}\right) = \boxed{2M_p \cdot \theta\left(\frac{2L}{2L-x}\right)}$$

$$EW = IW \rightarrow$$

$$\frac{q\,\delta}{2}\left(\frac{3L^2-2Lx}{2L-x}\right) = 2M_p\cdot\theta\left(\frac{2L}{2L-x}\right)$$

$$\theta = \frac{\delta}{x}$$

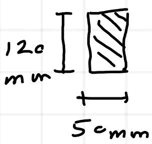
$$\frac{q\cdot\cancel{\delta}}{2}\left(\frac{3L^2-2Lx}{\cancel{2L-x}}\right) = 2M_p\cdot\frac{\cancel{\delta}}{x}\left(\frac{2L}{\cancel{2L-x}}\right)$$

$$\underset{(min)}{q} = \frac{4M_p\cdot 2L}{x\cdot(3L^2-2Lx)} = \frac{8M_p\cdot L}{\underbrace{3L^2x-2Lx^2}_{max}} \rightarrow L\cdot x(3L-2x)$$

$$\underset{(max)}{F(x)} = 3L^2x-2Lx^2 \qquad \frac{\partial F}{\partial x} = 3L^2-4Lx = 0 \Rightarrow \boxed{x = \frac{3}{4}L}$$

$$q_{min} = \frac{8M_p\,L}{L\cdot\frac{3}{4}L\left(3L-2\cdot\frac{3}{4}L\right)} = \frac{8M_p\,L}{\frac{3}{4}L^3\left(3-\frac{3}{2}\right)} = \frac{8M_p\,L}{\frac{3}{4}L^3\left(\frac{3}{2}\right)} = \frac{64M_p}{9L^2}$$

$$\boxed{q_{ult} = \frac{64}{9}\cdot\frac{M_p}{L^2}}$$




$\sigma_y = 250$ MPa

$$M_{pl} = \sigma_y\cdot\frac{bh^2}{4} = 250\ \text{MPa}\ \frac{50\ \text{mm}\times(120\ \text{mm})^2}{4} = 45\ \text{kNm}$$

$(\ell = 5\ \text{m})$

$$q_{ult} = \frac{64}{9}\cdot\frac{45\ \text{kNm}}{(5\ \text{m})^2} = 12.8\ \text{kN/m}$$