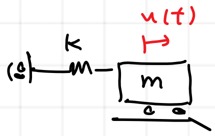
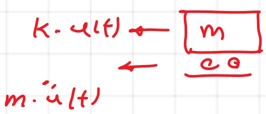
In the previous video, we covered the fundamental concepts of structural dynamics. In this video, we focus on solving a single-degree-of-freedom system that has a mass of 'm' and stiffness of 'k', without any damping. We provide an example and explore how altering the values and parameters can impact the results and initial conditions. These factors are crucial in determining the response of the structure to the applied load in free vibration.





$$\boxed{\frac{k}{m} = \omega^2}$$

$$\frac{m\ddot{u}(t) + k\,u(t) = 0}{m \neq 0} \rightarrow \ddot{u}(t) + \frac{k}{m} u(t) = 0$$

$$\ddot{u}(t) + \omega^2 u(t) = 0 \rightarrow r^2 + \omega^2 = 0 \Rightarrow r = \pm i\omega$$

$$\rightarrow u(t) = A \cos\omega t + B \sin\omega t \ , \quad \dot{u}(t) = -A\omega \sin\omega t + B\omega \cos\omega t$$

$u(t=0) = u_0 \qquad u(t=0) = A = u_0 \qquad \rightarrow \begin{cases} A = u_0 \\ B = \dfrac{\dot{u}_0}{\omega} \end{cases}$

$\dot{u}(t=0) = \dot{u}_0 \qquad \dot{u}(t=0) = B\omega = \dot{u}_0$

$$u(t) = \underbrace{u_0}_{A} \cos\omega t + \underbrace{\frac{\dot{u}_0}{\omega}}_{B} \sin\omega t$$

$$\beta = \tan^{-1}\left(\frac{u_0}{\dot{u}_0/\omega}\right) = \tan^{-1}\left(\frac{u_0 \cdot \omega}{\dot{u}_0}\right)$$

$$A^* = \sqrt{A^2 + B^2} = \sqrt{u_0^2 + \left(\frac{\dot{u}_0}{\omega}\right)^2}$$

$$Y = A \cos\alpha + B \sin\alpha$$

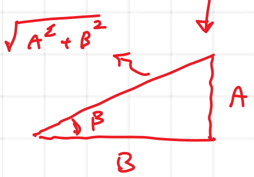
$$Y = B\left[\frac{A}{B}\cos\alpha + \sin\alpha\right] \qquad A/B = \tan\beta$$

$$Y = B\left[\tan\beta \cos\alpha + \sin\alpha\right]$$

$$Y = \frac{B}{\cos\beta}\left[\underbrace{\sin\beta \cdot \cos\alpha + \sin\alpha \cdot \cos\beta}_{\sin(\alpha+\beta)}\right]$$

$$y = \frac{B}{\cos\beta} \sin(\alpha + \beta)$$

$$\cos\beta = \frac{B}{\sqrt{A^2 + B^2}}$$



$$u(t) = A \sin(\omega t + \beta)$$

$$A = \sqrt{u_0^2 + \left(\frac{\dot{u}_0}{\omega}\right)^2}$$

$$\beta = \tan^{-1}\left(\frac{u_0\ \omega}{\dot{u}_0}\right)$$

$u_0 : L$

$\dot{u}_0 : LT^{-1}$

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{FL^{-1}}{M}}$

Natural frequency

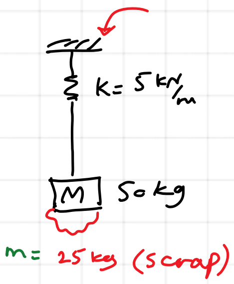
$F: M \cdot L\, T^{-2}$

$$\rightarrow \omega : \sqrt{\frac{\cancel{M} \cdot \cancel{L}\, T^{-2}\, \cancel{L}^{-1}}{\cancel{M}}} = T^{-1}$$

ω $\frac{\text{rad}}{\text{sec}}$

$$y = \underbrace{\sqrt{A^2 + B^2}}_{A:\ \text{amplitude}} \cdot \sin(\alpha + \underbrace{\beta}_{\text{Phase}})$$

$$T = \frac{2\pi}{\omega} \quad (s) \qquad f = \frac{1}{T} \quad (Hz)$$

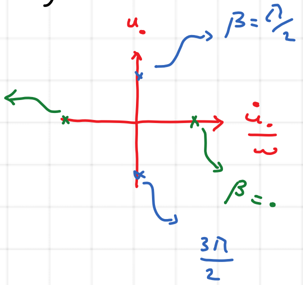


$$\delta_{(M+m)} = \frac{F}{K} = \frac{75\,kg \times 9.81\,m/s^2}{5\,kN/m} = \underline{147.15\,mm}$$

$$\delta_{(M)} = \frac{50\,kg \times 9.81\,m/s^2}{5\,kN/m} = \underline{98.1}\,mm$$

lets assume electricity is disconnected suddenly

$$\begin{cases} u_0 = 147.15 - 98.1\,mm = 49\,mm \quad (+) \\ \dot{u}_0 = 0 \end{cases}$$

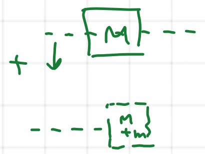


$$u(t) = A \sin(\omega t + \beta)$$

$$A = 49\,mm$$

$$A = \sqrt{u_0^2 + \left(\frac{\dot{u}_0}{\omega}\right)^2}$$

$$\beta = \pi/2$$



$$\beta = \tan^{-1}\left(\frac{u_0\,\omega}{\dot{u}_0}\right) \qquad \omega = \sqrt{\frac{K}{m}}$$

$$K = 5\,kN/m, \quad M = 50\,kg \;\rightarrow\; \omega = \sqrt{\frac{5000\,N/m}{50\,kg}} = 10\,\frac{rad}{sec}, \quad T = \frac{2\pi}{\omega} = 0.63\,sec, \quad f = 1.59\,(Hz)$$

$$u(t) = 49\,mm \cdot \sin\left(10\,t + \pi/2\right) = 49\,mm \cdot \cos(10\,t)$$