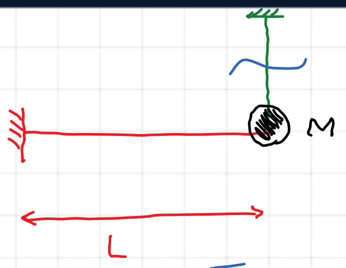
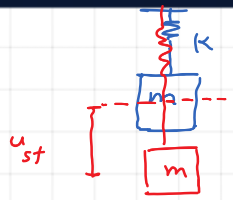
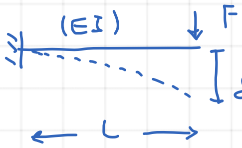
In the previous video, we focused on solving a single-degree-of-freedom system with a mass of 'm' and stiffness of 'k' without damping. We provided an example and explored how altering the values and parameters could impact the results and initial conditions.

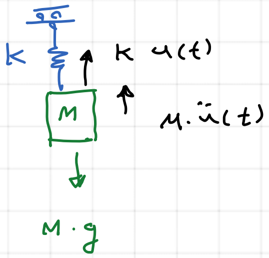
In this video, we will go through one example showing that the system starts to vibrate without applying external load or damping.







$\delta = \dfrac{F \cdot L^3}{3EI}$



$k = \dfrac{F}{\delta} = \dfrac{3EI}{L^3}$

$M \cdot \ddot{u}(t) + k\,u(t) = Mg$

$\ddot{u}(t) + \overset{\omega^2}{\dfrac{k}{m}}\, u(t) = g$

$\ddot{u}(t) + \omega^2 \cdot u(t) = g$

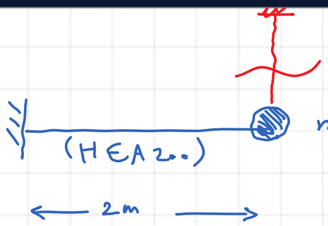
$u(t) = \underbrace{A \cos\omega t + B \sin\omega t}_{\text{general Sol}} + \underbrace{\dfrac{g}{\omega^2}}_{PS}$ $= k/m$ $\quad mg/k$

$\dot{u}(t) = -A \cdot \omega \sin\omega t + B\omega \cos\omega t$

$\left.\begin{array}{l} u_0 = 0 \\ \dot{u}_0 = 0 \end{array}\right\} \rightarrow \quad 0 = A + mg/k \rightarrow A = -mg/k$

$0 = 0 + B\omega \rightarrow B = 0$

$u(t) = -\dfrac{mg}{k} \cos\omega t + \dfrac{mg}{k} \Rightarrow \boxed{u(t) = \dfrac{mg}{k}(1 - \cos\omega t)}$



$m = 1\ \text{ton}$

HEA200 :

$E = 200\ GPa$

$I_y = 36.92 \times 10^6\ mm^4$

$L = 2m$

$$K = \frac{3EI}{L^3} = \frac{3 \times 200\ GPa \times 36.92 \times 10^6\ mm^4}{(2m)^3} = 2769\ N/mm$$

$$m = 1000\ kg \rightarrow \omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{2769\ N/mm}{1000\ kg}} = \sqrt{\frac{2769 \times 10^3\ N/m}{1000\ kg}} = 52.6\ \frac{rad}{s}$$

$$T = \frac{2\pi}{\omega} = 0.12\ sec$$

$$f = 8.3\ Hz$$

$$u(t) = \frac{mg}{K}(1 - \cos\omega t) = \frac{1000\ kg \times 9.81\ m/s^2}{2769 \times 10^3\ N/m} \cdot (1 - \cos\omega t) =$$

$$u(t) = \underset{m}{0.0035} \cdot (1 - \cos\omega t) = 3.5\ mm\ (1 - \underbrace{\cos\omega t}_{(-1)})$$

$$u_{max} = 3.5m\ (1+1) = 7\ mm$$

$$u_{st} = \frac{F}{K} = \frac{1000\ kg \times 9.81\ m/s^2}{2769 \times 10^3\ N/m} = 3.5\ mm$$