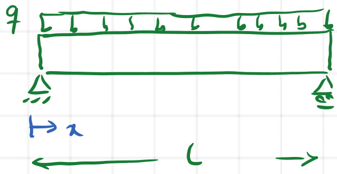
This video will teach us how to derive the total potential energy in an element subjected to bending resulting from the axial force. The final function helps one to calculate the buckling loads and stability of the compressive elements in buckling and post-buckling problems.



$$du = dx - dx \cdot \cos\theta$$

$$dx(1 - \cos\theta)$$

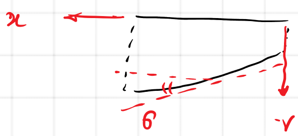
$$\cos\theta = \sqrt{1 - \sin^2\theta}$$

$$du = dx\left(1 - \sqrt{1 - \sin^2\theta}\right)$$

$\theta \to 0, \quad \sin\theta \simeq \theta$

$\theta \to 0$

$$\to du = dx\left(1 - \sqrt{1 - \theta^2}\right)$$



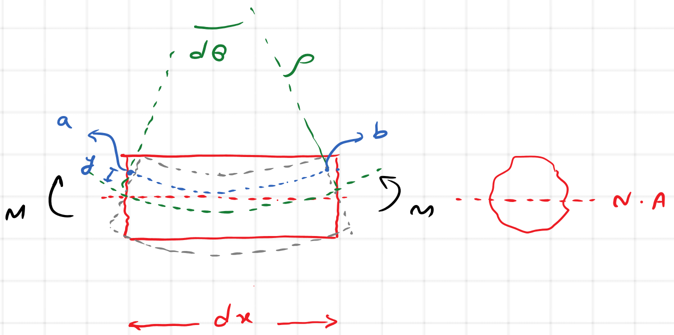
$$\theta = v'$$

$$du = dx \cdot \left(1 - \sqrt{1 - v'^2}\right) = dx\left(1 - \left(1 - \frac{1}{2} v'^2\right)\right) = \frac{1}{2} v'^2 \cdot dx$$

$$\sqrt{1 - v'^2} = 1 - \frac{1}{2} v'^2$$

$$d\bar{V} = -p \cdot du \Rightarrow dV = -P \cdot \frac{1}{2} \cdot v'^2 \cdot dx \Rightarrow$$

$$\bar{V} = -\frac{1}{2} P \int_0^L v'^2 \, dx$$



$$dx = \rho \cdot d\theta$$

$$ab = \underset{\rho - y}{\underline{\underline{\rho_{ab}}}} \cdot d\theta = (\rho - y)\, d\theta \Rightarrow \begin{cases} (ab)_1 = (\rho - y)\, d\theta \\ (ab)_0 = dx \end{cases}$$

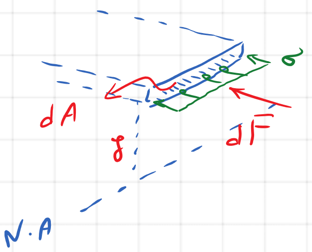
$$\delta_{ab} = (ab)_1 - (ab)_0 = (\rho - y)\, d\theta - dx$$

$$dx = \rho \cdot d\theta \Rightarrow d\theta = \frac{dx}{\rho}$$

$$\Bigg\} \Rightarrow \delta_{ab} = (\rho - y)\frac{dx}{\rho} - dx$$

$$\delta_{ab} = \cancel{\rho \frac{dx}{\rho}} - y\frac{dx}{\rho} - \cancel{dx} \Rightarrow \delta_{ab} = -\overset{K}{\left\{\frac{1}{\rho}\right\}}\, y \cdot dx$$

$$\delta_{ab} = -k \cdot y\, dx \rightarrow \boxed{d\delta = -k \cdot y \cdot dx} \quad (I)$$



$$\left.\begin{array}{l} dF = \sigma \cdot dA \\ \sigma = E \cdot \varepsilon \end{array}\right\} \Rightarrow dF = E \cdot \varepsilon \cdot dA$$

$$dW = \frac{dF \cdot d\delta}{2} = \frac{E \cdot \varepsilon \cdot dA \cdot d\delta}{2} = \frac{1}{2} \cdot E \cdot \frac{d\delta}{dx} \cdot dA \cdot d\delta$$

$$dW = \frac{1}{2} \cdot E \cdot (-k \cdot y) \cdot dA \cdot (-k \cdot y \cdot dx) = \frac{1}{2} \cdot E \cdot k^2 \cdot y^2 \cdot dA \cdot dx$$

$$W = \int_0^L \frac{1}{2} \cdot E \cdot k^2 \cdot y^2 dA \cdot dx = \int_0^L \frac{1}{2} \cdot E \cdot k \cdot \underbrace{\int_A y^2 dA}_{I} \cdot dx = \int_0^L \frac{1}{2} E \cdot k \cdot I\, dx$$

$$\Pi = W + V = \frac{1}{2}\int_0^L E \cdot I \cdot k^2 \, dx - \frac{1}{2} P \int v'^2 \, dx$$

$$k = -\frac{v''}{\sqrt{1-v'^2}} \cdot \frac{\sqrt{1+v'^2}}{\sqrt{1+v'^2}} = -\frac{v'' \cdot \sqrt{1+v'^2}}{\sqrt{1-v'^4}} = -v''\sqrt{1+v'^2}$$

$$v' \to 0$$

$$\sqrt{1+v'^2} = 1 + \frac{1}{2}v'^2 \Rightarrow k = -v'' \cdot \left(1 + \frac{1}{2}v'^2\right)$$

$$v' \to 0$$

$$\Pi = \frac{1}{2}\int_0^L E I \, v''^2 \cdot \left(1 + \frac{1}{2}v'^2\right)^2 dx - \frac{P}{2}\int v'^2 \, dx$$