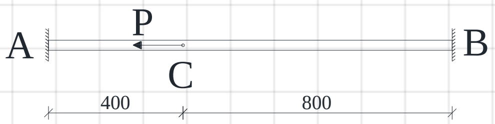
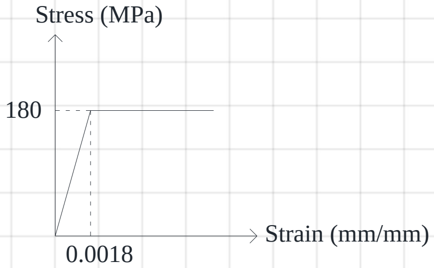
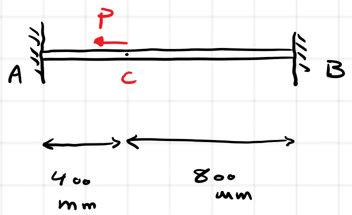
In the shown figure, a 30mm rod with an elastic perfectly plastic material is utilized. The yield stress and modulus of elasticity are 180MPa and 100GPa respectively. The rod is supported between fixed points A and B. The length of AB is 1200mm and at the distance of 400mm from support A is subjected to a horizontal load P.

a) Which element would yield first while the force P is increased?
b) Determine the corresponding load that one element would yield.
c) Sketch the force deformation of point C with the respect to the force increment.
d) Determine the maximum plastic deformation of element AC when the maximum yield force is applied.





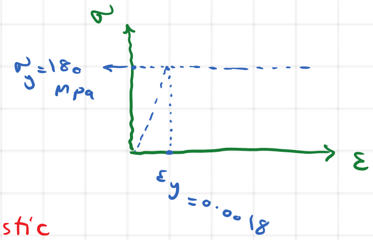


$d = 30\,mm$

$E = 100\,GPa$

$\sigma_y = 180\,MPa$

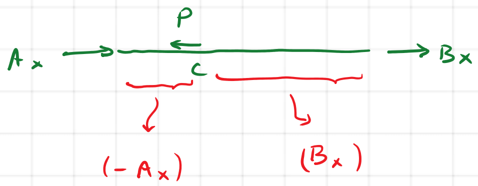
Elastic, perfectly plastic



$$\varepsilon_y = \frac{\sigma_y}{E} = 0.0018$$

$$F_y = \sigma_y \cdot A = 180\,MPa \cdot \left(\frac{\pi \times (30\,mm)^2}{4}\right) = 127\,kN$$

$$(\delta_c)_{AC} = (\delta_c)_{BC}$$



$$\delta = \frac{F \cdot L}{A\,E}$$

$$\overset{+}{\rightarrow}\ \Sigma F_x = 0 \rightarrow A_x + B_x = P \quad (*)$$

$$\frac{A_x \cdot (400\,mm)}{A_{AC} \cdot E_{AC}} = \frac{B_x \cdot (800\,mm)}{A_{BC} \cdot E_{BC}} \rightarrow \text{in elastic phase: } \boxed{A_x = 2B_x} \quad (I)$$

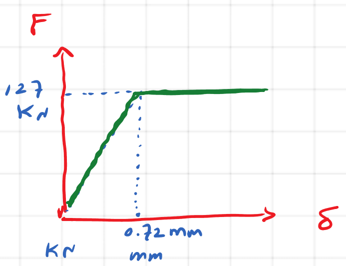
$F_y = 127\,kN \rightarrow B_x$ is going to yield first:

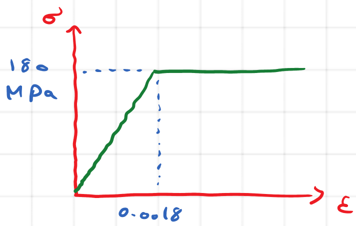
$B_x = 127\,kN \xrightarrow{(I)} A_x = 2 \cdot (127\,kN) = 254\,kN > F_y = 127\,kN$ ✗

$A_x = 127\,kN \xrightarrow{(I)} B_x = \frac{127\,kN}{2} = 63.5\,kN < F_y = 127\,kN$ (ok)

$$(*) \rightarrow P = A_x + B_x = 127 + 63.5 = 190.5\,kN$$

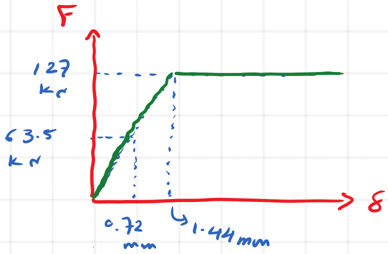
(AC)

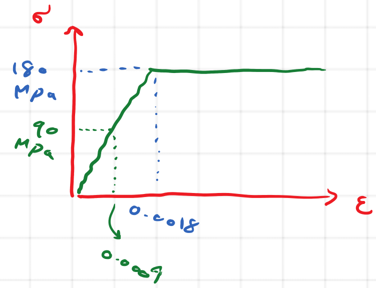




$$\delta = \frac{F \cdot L}{A\,E} = \frac{127\,kN \times 400\,mm}{\frac{\pi \times (30\,mm)^2}{4} \times 100\,GPa} = 0.72\,mm\ ,\quad \varepsilon_y \cdot L_{AC} = 0.0018 \times 400\,mm = 0.72\,mm$$

(BC)





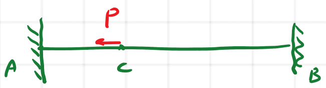
$P = 190.5\ kN \rightarrow$

$F_{BC} = 63.5\,kN \rightarrow \delta = \frac{F \cdot L}{A\,E} = \frac{63.5\,kN \times 800\,mm}{\frac{\pi \times (30\,mm)^2}{4} \times 100\,GPa} = 0.72\ mm \rightarrow \varepsilon_{BC} = \frac{\delta}{L_{BC}} = \frac{0.72\,mm}{800\,mm} = 0.0009$

$\sigma = \frac{F}{A} = \frac{63.5\,kN}{\frac{\pi \times (30\,mm)^2}{4}} = 89.8 \simeq 90\ MPa$

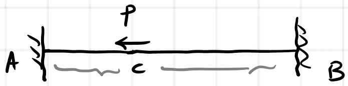
$\delta_y = \varepsilon_y \cdot L = 0.0018 \times 800\,mm = 1.44\ mm$

For force P



$\cdot < P < 190.5 \rightarrow$ AC becomes plastic (compression)

BC still in elastic phase.



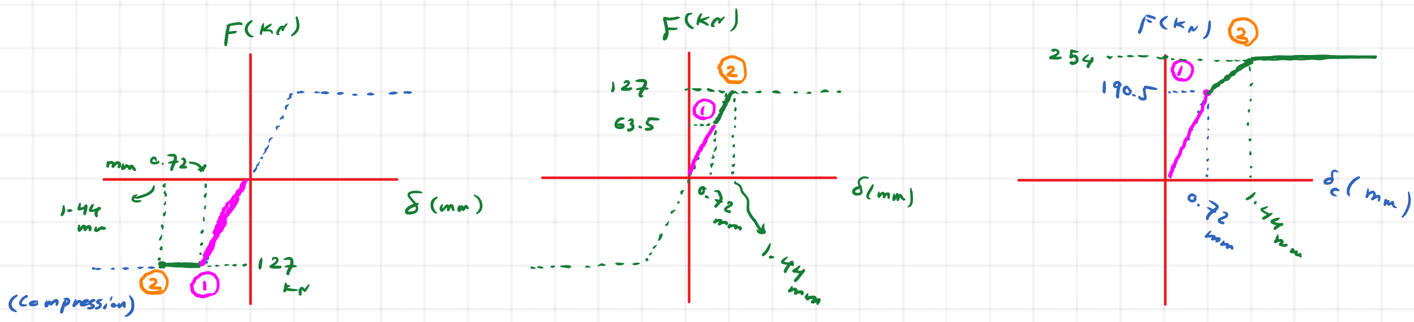
$P_y = (F_y)_{AC} + (F_y)_{BC}$

$F_{y\,(AC)}$: 127 kN

$F_{y\,(BC)}$: 63.5 kN; 127 kN − 63.5 kN = 63.5 kN

1st approach → $\underbrace{190.5}_{kN} + \underbrace{(127 - 63.5)}_{kN} = 254\ kN$

2nd approach → $P_y = F_{y_{AC}} + F_{y_{BC}} = 127\,kN + 127\,kN = 254\ kN$



(AC) (BC) P

① → $P = 190.5$ kN

② → $P = P_y = 254$ kN

1st example of the load:

$P = 160$ kN → if the load is removed what is the stress and strain in the elements?

2nd example of the load:

$P = 220$ kN → if the load is removed what is the stress, strain, permanent deformation of the elemnts?