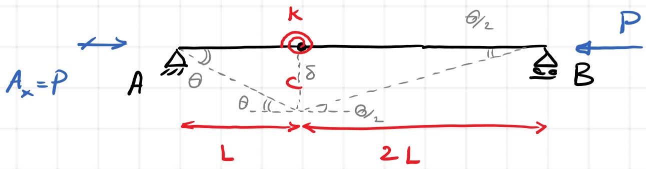
Two rigid elements are connected with a hinge at point *c* with a rotational spring with a constant stiffness of *k*. The entire system is under a compressive load of P at the end B, supported by a roller. By increasing the compressive load, the system will buckle.

a) Determine the total potential energy of the system.
b) Determine the primary and secondary paths.
c) Check the paths in terms of being stable or unstable.



$$\Delta HB = L - L\cos\theta + 2L - 2L\cos(\theta/2)$$

$$\Delta HB = L\left[1-\cos\theta + 2 - 2\cos\left(\frac{\theta}{2}\right)\right]$$

$$|\theta| = \theta + \theta/2 = 3\theta/2$$

$$V = -PL\left[1-\cos\theta + 2 - 2\cos\left(\frac{\theta}{2}\right)\right]$$

$$W = \frac{1}{2}k\left(\frac{3\theta}{2}\right)^2 = \frac{9}{8}k\theta^2$$

$$\Pi(\theta) = W + V = \frac{9}{8}k\theta^2 - PL\left[1-\cos\theta + 2 - 2\cos\left(\frac{\theta}{2}\right)\right]$$

1)

$$\theta \to 0 \Rightarrow 1-\cos\theta \simeq \frac{1}{2}\theta^2$$

$$\to \Pi(\theta) = \frac{9}{8}k\theta^2 - PL\left[\frac{1}{2}\theta^2 + 2\,\frac{1}{2}\left(\frac{\theta}{2}\right)^2\right] = \frac{9}{8}k\theta^2 - \frac{3}{4}PL\theta^2$$

$$\frac{1}{2}\theta^2 + \frac{1}{4}\theta^2$$

$$\frac{3}{4}\theta^2$$

$$\frac{\partial\Pi}{\partial\theta} = 0 \Rightarrow \frac{9}{4}k\theta - \frac{3}{2}PL\theta = 0 \Rightarrow \begin{cases}\theta = 0\\ P = \frac{3}{2}\frac{k}{L} \Rightarrow P_0 = \frac{3}{2}\frac{k}{L}\end{cases}$$

2)

$$\Pi(\theta) = W + V = \frac{9}{8}k\theta^2 - PL\left[1-\cos\theta + 2 - 2\cos\left(\frac{\theta}{2}\right)\right]$$

$$\frac{\partial\Pi}{\partial\theta} = 0 \Rightarrow \frac{9}{4}k\theta - PL\left[\sin\theta + \sin\left(\frac{\theta}{2}\right)\right] = 0 \Rightarrow \boxed{\theta = 0} \to \text{Primary path}$$

$$\left(PL = \frac{9}{4}k\theta \cdot \frac{1}{\sin\theta + \sin\frac{\theta}{2}}\right) : L \Rightarrow \left(P_{cr} = \frac{9}{4}\cdot\frac{k}{L}\cdot\frac{\theta}{\sin\theta + \sin\frac{\theta}{2}}\right) : P_0\Big|_{\frac{3}{2}\frac{k}{L}}$$

$$\to \lambda \left(\frac{P_{cr}}{P_0}\right) = \frac{3}{2}\,\frac{\theta}{\sin\theta + \sin\frac{\theta}{2}} \Rightarrow \boxed{\lambda = \frac{3}{2}\cdot\frac{\theta}{\sin\theta + \sin\frac{\theta}{2}}} \Rightarrow \text{secondary Path.}$$

Primary path $\theta = 0$

Secondary path $\lambda = \frac{3}{2} \dfrac{\theta}{\sin\theta + \sin\frac{\theta}{2}}$

$\frac{\partial \pi}{\partial \theta} = 0 \Rightarrow \frac{9}{4} k\theta - PL\left[\sin\theta + \sin\left(\frac{\theta}{2}\right)\right]$

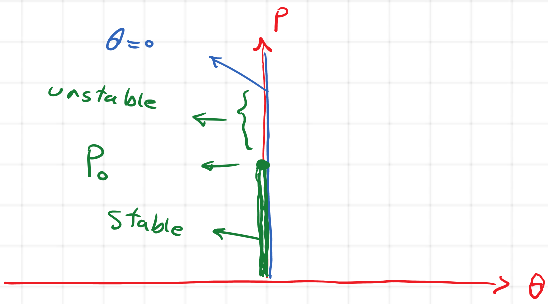
$\boxed{\frac{\partial^2 \pi}{\partial \theta^2} > 0} \rightarrow$ stable

$$\frac{\partial^2 \pi}{\partial \theta^2} = \frac{9}{4}k - PL\left[\cos\theta + \frac{1}{2}\cos\frac{\theta}{2}\right] > 0$$

$$\theta = 0 \Rightarrow \frac{9}{4}k - PL\left[1 + \frac{1}{2}\right] = \frac{9}{4}k - PL \cdot \frac{3}{2} > 0 \Rightarrow P < \underbrace{\frac{3}{2} \cdot \frac{k}{L}}_{P_0} \Rightarrow \text{stable}$$

$\boxed{\lambda < 1}$

$P_0 = \frac{3}{2}\frac{K}{L}$



Secondary path: $\lambda = \frac{3}{2} \dfrac{\theta}{\sin\theta + \sin\frac{\theta}{2}}$

$$\frac{\partial^2 \pi}{\partial \theta^2} = \frac{9}{4}k - PL\left[\cos\theta + \frac{1}{2}\cos\frac{\theta}{2}\right] > 0 \Rightarrow P < \underbrace{\frac{9}{4} \cdot \frac{k}{L} \cdot \frac{1}{\cos\theta + \frac{1}{2}\cos\frac{\theta}{2}}}_{P_0}$$

$P_0 = \frac{3}{2}\frac{K}{L}$

$$\underbrace{\frac{P}{P_0}}_{\lambda} < \frac{3}{2} \cdot \frac{1}{\cos\theta + \frac{1}{2}\cos\frac{\theta}{2}} \Rightarrow \lambda \leq \frac{3}{2} \cdot \frac{1}{\cos\theta + \frac{1}{2}\cos\frac{\theta}{2}} \Rightarrow \underset{\sim}{\text{stable}}$$