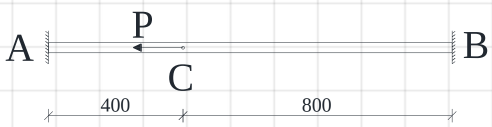
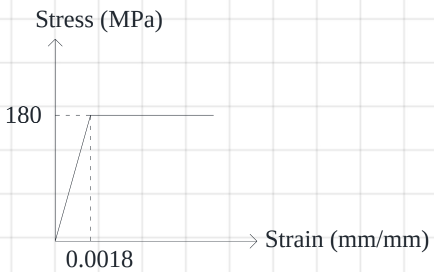
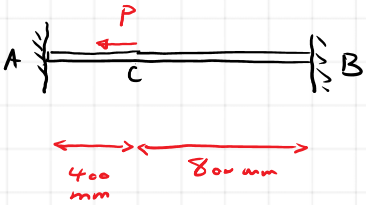
In the previous video, we determined the yielding load of P to be 254kN. Also, the load of P can be increased up to 190.5kN before one element, AC, becomes plastic. For the same example:

a) If load P is increased to 160kN and is removed from the system, what is the stress in the rods after the load is removed?
b) If the load is increased to 220kN and removed, determine the permanent deformation of point C as well as residual stresses in each element.



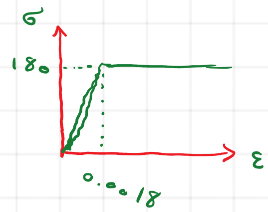


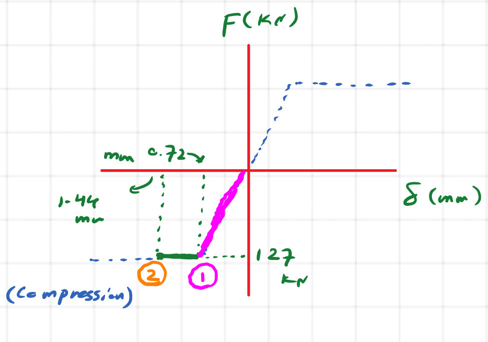


$E = 100$ GPa

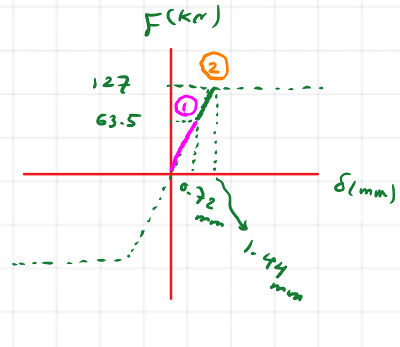
$\sigma_y = 180$ MPa

$d = 30$ mm

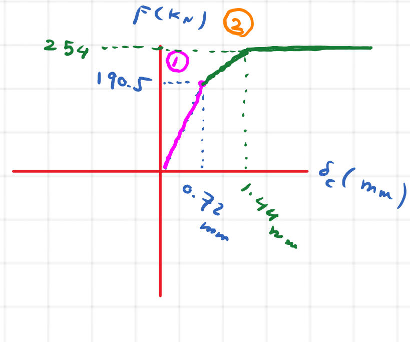




(AC)



(BC)



P

$P = 160\,kN < 190.5\,kN \rightarrow$ Two rods are loaded less than their yielding load.

As a result the system is elastic.

$$\begin{cases} A_x + B_x = P \\ A_x = 2B_x \end{cases} \qquad \left.\begin{aligned} A_x + B_x &= 160 \\ A_x &= 2B_x \end{aligned}\right\} \Rightarrow 3B_x = 160 \Rightarrow \boxed{B_x = 53.3\,kN}$$

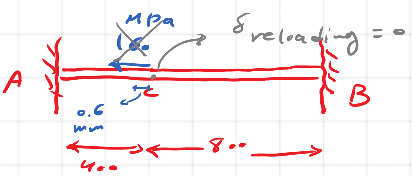
$$A_x = 106.6\,kN$$

$$A_x,\ B_x < 127\,kN = F_y \longrightarrow \sigma_{AC} = \frac{A_x}{A} = \frac{106.6\,kN}{\frac{\pi \times (30\,mm)^2}{4}} = 151\ MPa \leqslant 180\ MPa$$

$$\sigma_{BC} = \frac{B_x}{A} = \frac{53.3\,kN}{\frac{\pi \times (30\,mm)^2}{4}} = 75.5\ MPa \leqslant 180\ MPa$$

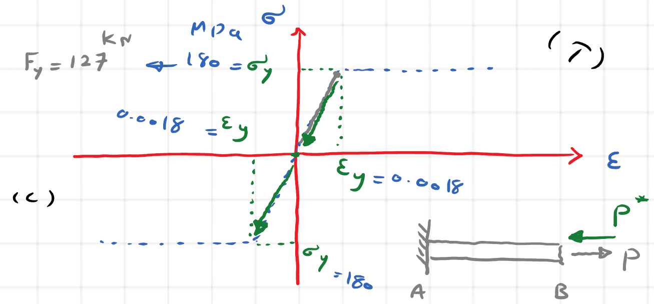
$$\varepsilon_{AC} = \frac{\sigma_{AC}}{E} = \frac{151\ MPa}{100\ GPa} = 0.0015 < \varepsilon_y = 0.0018 \rightarrow \delta_{AC} = \varepsilon \cdot L = 0.0015 \times 400\,mm = 0.6\,mm$$

$$\varepsilon_{BC} = \frac{\sigma_{BC}}{E} = \frac{75.5\,MPa}{100\ GPa} = 0.00075 \leqslant \varepsilon_y = 0.0018 \rightarrow \delta_{BC} = \varepsilon \cdot L = 0.00075 \times 800 = 0.6\,mm$$

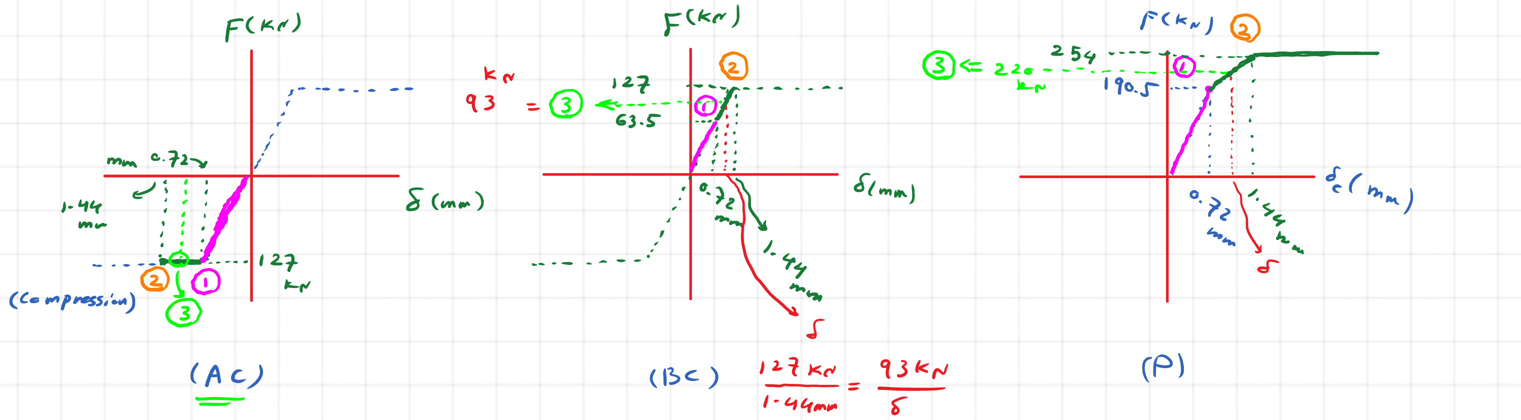


After unloading $\begin{cases} \sigma_{AC} = \sigma_{BC} = 0 \\ \delta_C = 0 \end{cases}$

$P = 220 \text{ kN} \rightarrow$



in the loading from zero position is 127 kN then in reverse loading the element will be elastic with the capacity of $2 \times 127 \text{ kN} = 254 \text{ kN}$



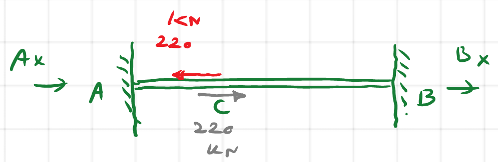
$$\frac{127 \text{ kN}}{1.44 \text{ mm}} = \frac{93 \text{ kN}}{\delta}$$

$$\delta = 1.05 \text{ mm}$$

$$\frac{(\text{kN})\ 254 - 190.5}{(\text{mm})\ 1.44 - 0.72} = \frac{220 - 190.5 \ (\text{kN})}{\delta - 0.72 \ (\text{mm})}$$

$$\delta = 1.05 \text{ mm}$$

$$\boxed{P = 220 \text{ kN}}$$



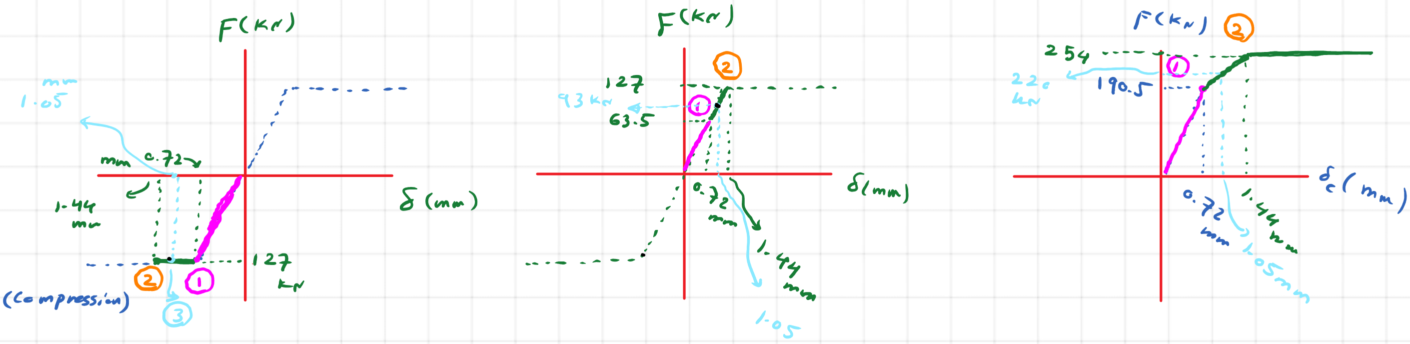
loading $P = 220 \text{ kN} \rightarrow$ Elastic relation betwee A_x & B_x is not valid any more.

$$\rightarrow \begin{cases} A_x + B_x = P \\ A_x = 2B_x \end{cases} \rightarrow 3B_x = 220 \text{ kN} \rightarrow \begin{cases} B_x = 73.3 \text{ kN} \leqslant 127 \text{ kN} \ (\text{ok}) \\ A_x = 146.6 \text{ kN} \nleqslant 127 \text{ kN} \ (\text{Not ok}) \end{cases}$$

$P = 220 \text{ kN} \rightarrow$ the system is not elastic for all members.

$$\left.\begin{aligned} A_x &= 127 \text{ kN} \\ A_x + B_x &= P = 220 \text{ kN} \end{aligned}\right\} \rightarrow B_x = 220 \text{ kN} - 127 \text{ kN} = 93 \text{ kN} \leqslant 127 \text{ kN} \quad (\text{ok})$$

$$\delta_c = \frac{F_{BC} \cdot L}{A \cdot E} = \frac{93 \text{ kN} \times 800 \text{ mm}}{\frac{\pi \times (30 \text{ mm})^2}{4} \times 100 \text{ Gpa}} = 1.05 \text{ mm}$$



(AC) (BC) P

reverse loading → Now the capacity of AC → $2 \times 127\,kN = 254\,kN$

the capacity of BC → $93\,kN + 127\,kN = 220\,kN$

Elastic analysis is valid → $\begin{cases} A_x = 2B_x \\ A_x + B_x = 220\,kN \end{cases}$ → $3B_x = 220\,kN \Rightarrow B_x = 73.3\,kN$ < Elastic capacity 220 kN (OK)

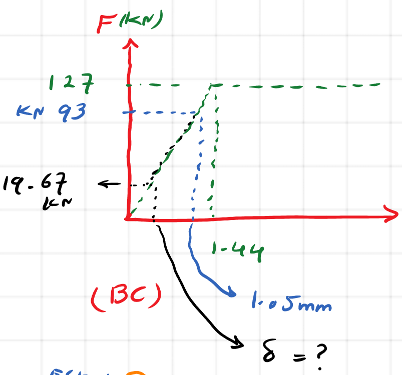
$A_x = 2B_x = 146.6\,kN$ < Elastic capacity 254 (OK)

loading $F_{AC} = -127\,kN$ reverse loading $F_{AC} = 146.6\,kN$

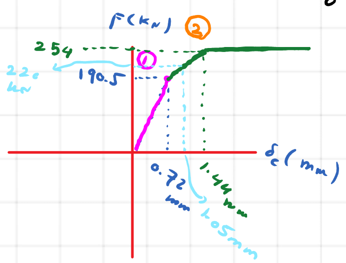
$F_{BC} = +93\,kN$ $F_{BC} = -73.3\,kN$

$$F_{AC(Final)} = -127 + 146.6 = 19.6\,kN$$

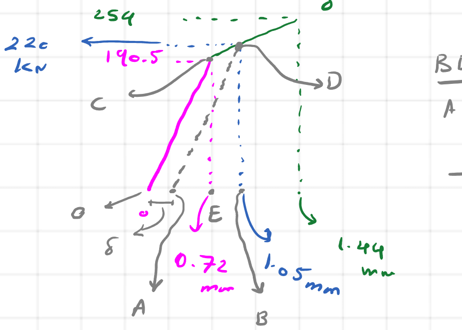
$$F_{BC(Final)} = +93\,kN - 73.3\,kN = 19.7\,kN$$



$$\frac{127\,(kN)}{1.44\,(mm)} = \frac{19.67\,(kN)}{\delta} \Rightarrow \delta = 0.22\,mm$$



P



$$\frac{BD}{AB} = \frac{CE}{OE} \Rightarrow \frac{220}{AB} = \frac{190.5}{0.72}$$

$\rightarrow AB = 0.832\,mm$

$OA = \delta = OB - AB = 1.05\,mm - 0.832\,mm$

$$\boxed{\delta = 0.22\,mm}$$

$F_{AC} = 19.67\ kN \rightarrow \sigma = \dfrac{19.67\ kN}{\frac{\pi (30mm)^2}{4}} = 28\ Mpa$

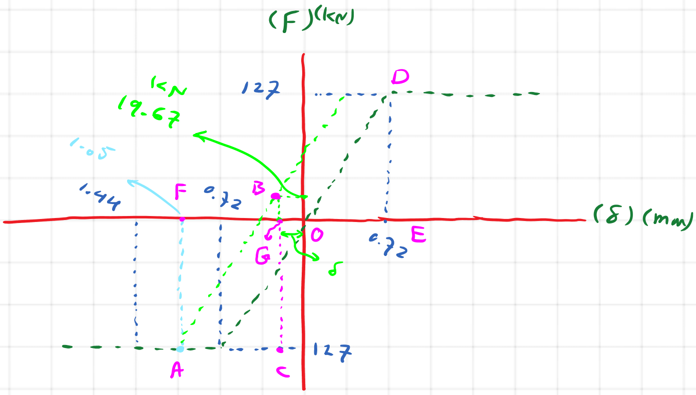
$F_{BC} = 19.67\ kN \rightarrow \sigma = 28\ Mpa$

loading $\rightarrow \begin{cases} F_{AC} = 127\ kN \longrightarrow \sigma_{AC} = -180\ Mpa \\ F_{BC} = 93\ kN \longrightarrow \sigma_{BC} = \dfrac{93\ kN}{\frac{\pi \times (30mm)^2}{4}} = 131.6\ Mpa \end{cases}$

reloading $\rightarrow \begin{cases} F_{AC} = 146.6\ kN \longrightarrow \sigma_{AC} = 207.5\ Mpa \ \oplus \\ F_{BC} = 73.3\ kN \longrightarrow \sigma_{BC} = 103.75\ Mpa \ \ominus \end{cases}$

$\sigma_{Final(AC)} = \overset{Mpa}{-180} + \overset{Mpa}{207.5} = 27.5\ Mpa \simeq 28\ Mpa$

$\sigma_{Final(BC)} = \overset{Mpa}{131.6} - \overset{Mpa}{103.75} = 26.85\ Mpa \simeq 27\ Mpa$



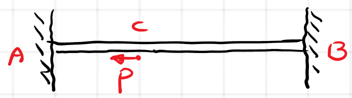
$\dfrac{BC}{AC} = \dfrac{DE}{OE}$

$\dfrac{127 + 19.67}{AC} = \dfrac{127}{0.72}$

$AC = 0.83$

$\delta = OF - \underbrace{FG}_{AC} = OF - AC = 1.05 - 0.83 = 0.22\ mm$

$0 < P < \overset{kN}{190.5}$



$P = 190.5 \rightarrow \begin{cases} AC = \overset{kN}{127} \\ BC = 63.5 \end{cases}$

$190.5 < P < \overset{kN}{220} \rightarrow \begin{cases} AC = \overset{kN}{127} \\ BC = 93\ kN \end{cases}$

$P \xrightarrow[kN]{220} \quad \xrightarrow[kN]{0} \quad \rightarrow F_{AC} = F_{BC} = 19.67\ kN \rightarrow \boxed{\delta_c = 0.22\ mm}$