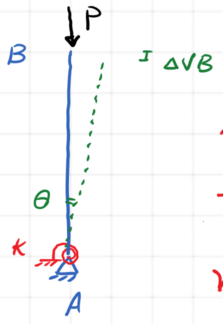
One rigid element with a length of l is subjected to a compressive load at its tip. At the bottom, the element is supported by a simple hinge and a rotational spring with constant k.

a) Determine the total potential energy of the system.
b) Determine the buckling load as a function of rotational angle.
c) Find the bifurcation point.
d) Determine the primary and secondary paths.
e) Check the paths in terms of being stable or unstable.



$\Delta V_B = L - L\cos\theta$

$V = -PL(1-\cos\theta)$

$W = \frac{1}{2}k\cdot\theta^2$

$\pi(\theta) = W + V = \frac{1}{2}k\theta^2 - PL(1-\cos\theta)$

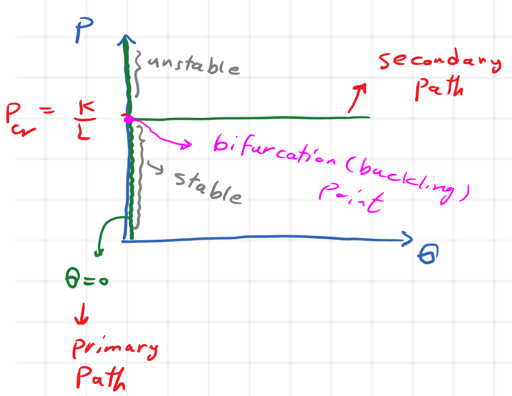
$\theta \to 0 \Rightarrow 1-\cos\theta \simeq \frac{1}{2}\theta^2$

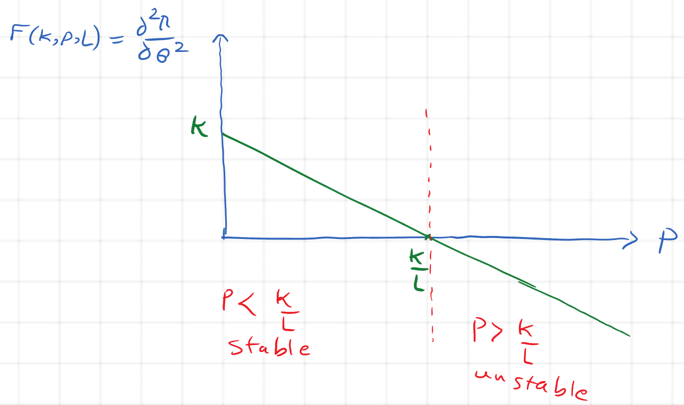
$\pi(\theta) = \frac{1}{2}k\theta^2 - PL\frac{\theta^2}{2}$

$\frac{\partial\pi}{\partial\theta} = k\theta - PL\theta = 0 \Rightarrow P_{cr} = \frac{k}{L} = P_0 \Rightarrow \boxed{\lambda = 1}$

$\frac{\partial\pi^2}{\partial\theta^2} = k - PL > 0 \Rightarrow P < \frac{k}{L} \Rightarrow \frac{P}{P_{cr}} < 1 \Rightarrow \boxed{\lambda < 1}$

$k\theta - PL\theta = \theta(k - PL) = 0 \Rightarrow \begin{cases} P = \frac{k}{L} \rightarrow \text{secondary path} \\ \theta = 0 \rightarrow \text{Primary path} \end{cases}$





$\frac{\partial^2\pi}{\partial\theta^2} = k - PL \Rightarrow F(k,P,L) = k - PL$

$\pi(\theta) = \frac{1}{2}k\theta^2 - PL(1-\cos\theta)$

$\frac{\partial\pi}{\partial\theta} = k\theta - PL\sin\theta = 0 \Rightarrow P = \frac{k}{L}\cdot\frac{\theta}{\sin\theta} \Rightarrow$ critical load

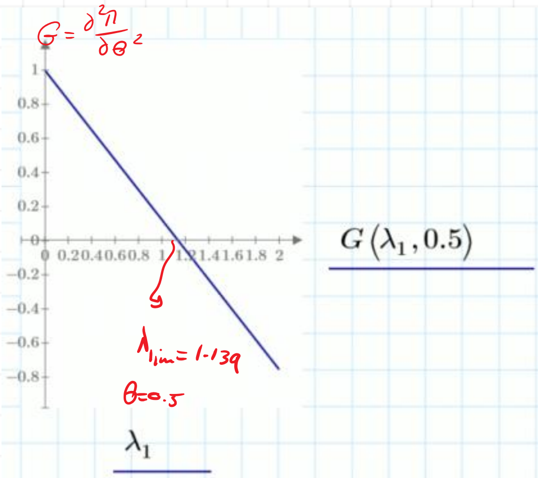
$\frac{\partial^2\pi}{\partial\theta^2} = \underbrace{k - PL\cos\theta}_{F(k,P,l,\theta)} > 0 \Rightarrow P < \frac{k}{L}\cdot\frac{1}{\cos\theta} \Rightarrow$ stable

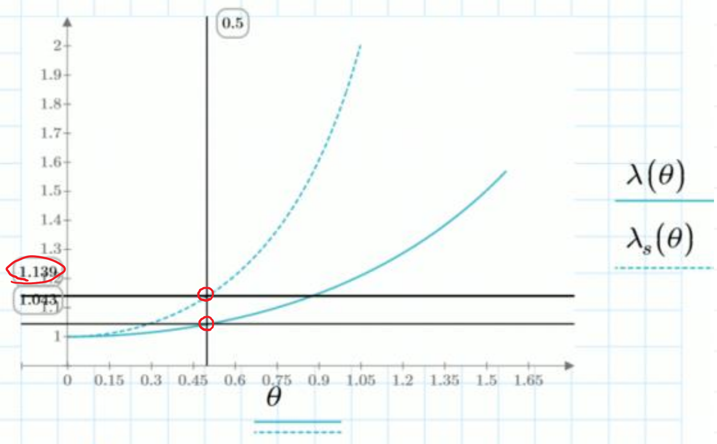
$$\left[F(k,p,l,\theta) = k - pL\cos\theta\right] : k \Rightarrow G = 1 - \frac{P}{k/L}\cos\theta = 1 - \lambda\cos\theta$$

$$\left(P_{cr} = \frac{k}{L}\,\frac{\theta}{\sin\theta}\right) : \frac{k}{L} \Rightarrow \frac{P_{cr}}{P_{cr_0}} = \lambda = \frac{\theta}{\sin\theta}$$

$$P_{cr_0} = \frac{k}{L}$$

$$k - PL\cos\theta > 0 \Rightarrow P < \frac{k}{L}\cdot\frac{1}{\cos\theta} \Rightarrow \lambda < \frac{1}{\cos\theta}$$





$1 - \lambda\cos\theta = 0$

$\theta = 0.5$

$\lambda_{limit} = 1.139$ If $\lambda < 1.139$

$\rightarrow$ stable

$$\theta = 0.5 \Rightarrow \lambda = \frac{\theta}{\sin\theta} = 1.043 < 1.139$$

stable