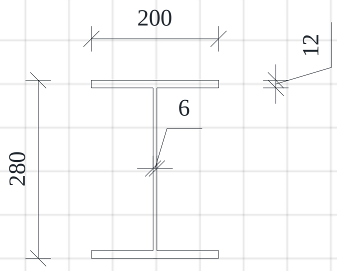
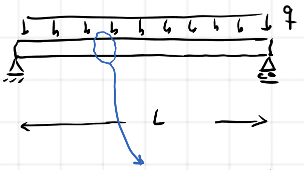
In the other Video, we derived the relationship between the strain and curvature of a beam under bending moment. Also, in the former Video, we derived from calculating Elastic and Plastic load for a rigid bar supported by several deformable bars.

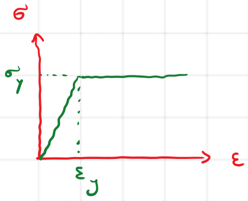
In this example, we will learn how the elastic axial stress in a beam is determined and the axial stress due to bending distribution. Moreover, the elastic section modulus is introduced. The fully plastic behavior of a cross-section is reviewed with two examples. In the end, a symmetrical I-Beam is studied in terms of elastic bending moment, plastic bending moment, and partial bending moment calculation. The descriptions of the questions in the Video are as follows:

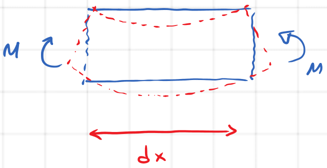
a) Determine a rectangle beam's elastic and plastic section modulus, b as width and height.
b) If $b = 120mm$, and $h = 200mm$ determine the elastic and plastic bending moment.
c) If the bending moment applied to the cross-section is $250kN.m$, what is the plastic depth of the rectangular section?
d) Determine the elastic and plastic section modulus for the given I-Beam section.
e) For the I-Beam, determine the elastic and plastic moment.
f) Determine the required bending moment so that only flanges become plastic.
g) If the applied bending moment is $180kN.m$, what is the plastic depth of the cross-section?

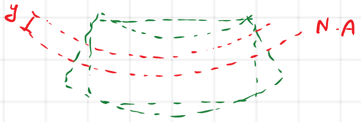
Assume the material is elastic and perfectly plastic with a yield stress of $250MPa$.





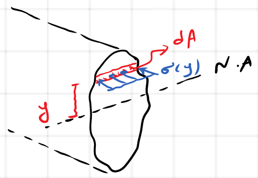






$$d\delta(y) = -k \cdot y \cdot dx$$

$$\varepsilon = \frac{\delta}{L} = \frac{d\delta(y)}{dx} = -k \cdot y \quad (\mathrm{I})$$



$$dM = dF \cdot y$$
$$dF = \sigma(y) \cdot dA$$
$$\Big\} \; dM = \sigma(y) \cdot y \cdot dA$$

Elastic phase : $\boxed{\sigma = E \cdot \varepsilon}$ (II) $\rightarrow dM = E \cdot \varepsilon(y) \cdot y \cdot dA$

$$(\mathrm{I}) \rightarrow dM = E \cdot -k \cdot y \cdot y \cdot dA$$

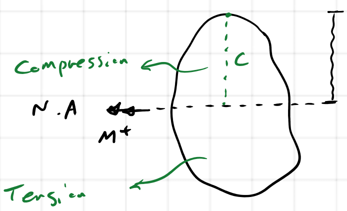
$$dM = -E \cdot k \cdot y^2 dA$$

$$M = \int -E \cdot k \cdot y^2 \cdot dA = -E \cdot k \overbrace{\int_A y^2 dA}^{I} \rightarrow M = E \cdot k \cdot I$$

$$k = \frac{1}{\rho} \rightarrow M = \frac{E \cdot I}{\rho} \Rightarrow \boxed{\frac{1}{\rho} = \frac{M}{EI}}$$

$$(\mathrm{II}) \rightarrow \sigma = E \cdot \overset{(\mathrm{I})}{\varepsilon} = E \cdot -k \cdot y = -k \cdot E \cdot y = -\frac{1}{\rho} \cdot E \cdot y = -\frac{M}{\cancel{E}I} \cdot \cancel{E} \cdot y$$

$$\rightarrow \boxed{\sigma(y) = -\frac{M \cdot y}{I}}$$

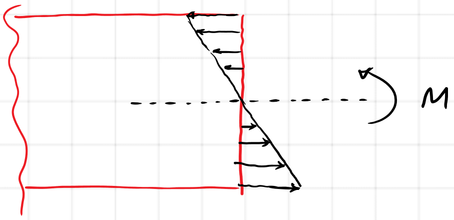


$$\sigma \rightarrow Max \rightarrow y \rightarrow max \qquad \sigma = -\frac{M \cdot y}{I}$$

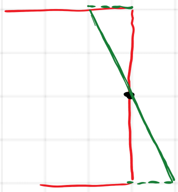
$$\sigma_{max} = -\frac{M \cdot y_{max}}{I} \rightarrow \sigma_{max} = \frac{M \cdot c}{I} = \frac{M}{I/c}$$

section modulus

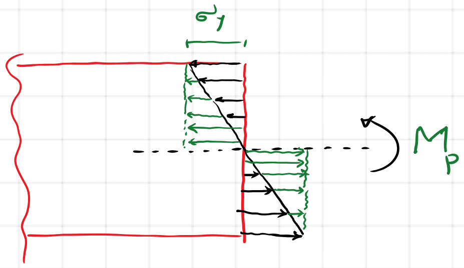
$$W = S = I/c \rightarrow \boxed{W_{el} = \frac{I}{c}}$$

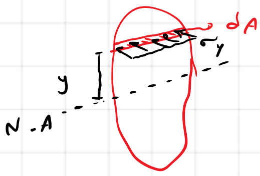


$$\sigma = -\frac{M \cdot y}{I}$$



$$\sigma_{max} = \frac{M}{W_{el}} \rightarrow \sigma_y = \frac{M_y}{W_{el}} \Rightarrow \boxed{M_y = \sigma_y \, W_{el}}$$





$$dF(y) = \sigma_y \cdot dA$$

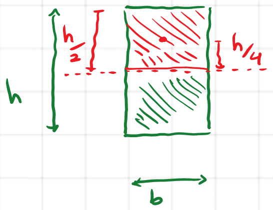
$$dM = dF(y) \cdot y$$

$$\rightarrow dM = \sigma_y \cdot y \cdot dA \rightarrow M_p = \int \sigma_y \cdot y \cdot dA = \sigma_y \overbrace{\int y \cdot dA}^{W_{pl}}$$

$$M_{el} = \sigma_y \cdot W_{el} \qquad W_{el} = I/c$$

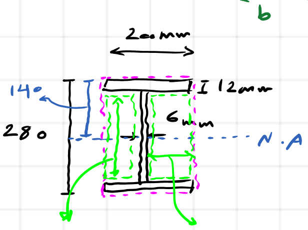
$$M_p = \sigma_y \cdot W_{pl} \qquad W_p = \int y \cdot dA = \Sigma\, y_i A_i$$

Example :



$$I = bh^3/12 \ , \ c = \frac{h}{2} \ , \ W_{el} = \frac{I}{c} = \frac{bh^2}{6}$$

$$W_p = b \cdot \frac{h}{2} \cdot \frac{h}{4} + b\,\frac{h}{2} \cdot \frac{h}{4} = \frac{bh^2}{4}$$

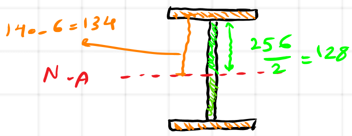


$$I = \frac{200 \times 280^3}{12} - 2 \times \frac{97 \times 256^3}{12} = 9.5 \times 10^7 \ mm^4$$

$$c = 140_{mm} \rightarrow W_{el} = \frac{I}{c} = \frac{9.5 \times 10^7 \ mm^4}{140 \ mm} = 6.76 \times 10^5 \ mm^3$$

$$\frac{280 - 2\times12}{256} \qquad \frac{200-6}{2} = 97$$

$$W_p = \Sigma A_i \cdot y_i = 200 \times 12 \times 134 \times 2 + 128 \times 6 \times \frac{128}{2} \times 2 = 7.41 \times 10^5 \ mm^3$$



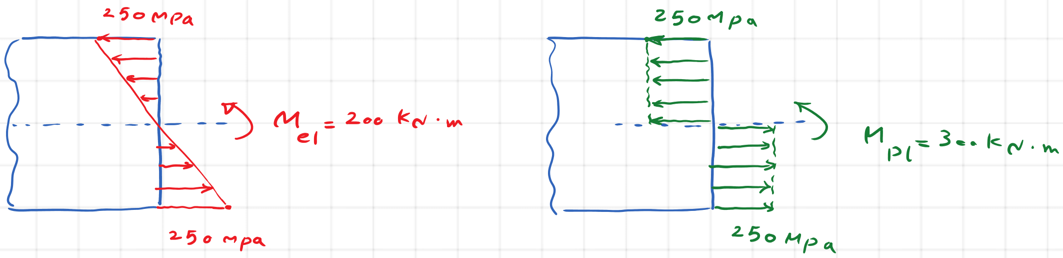
$$\begin{cases} b = 120\ mm \\ h = 200\ mm \end{cases} \qquad W_{el} = \frac{bh^2}{6} = 800000\ mm^3$$

$$W_{pl} = \frac{bh^2}{4} = 1200000\ mm^3$$

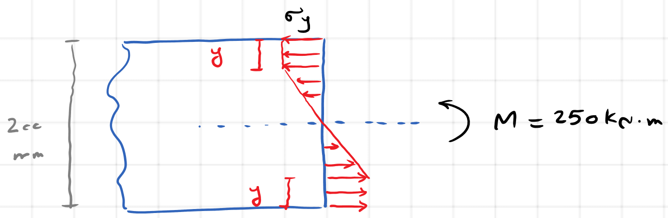
$(\sigma_y = 250\ Mpa)$

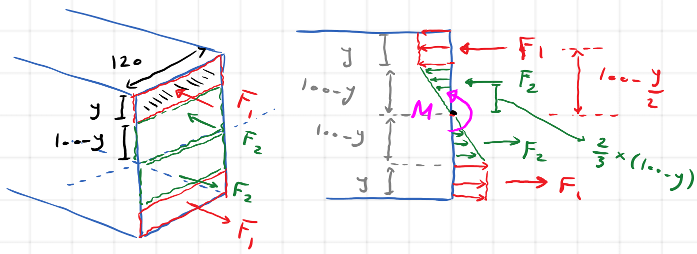
$$M_{el} = \sigma_y \cdot W_{el} = 250\ Mpa \times 800000\ mm^3 = 200\ kN \cdot m$$

$$M_{pl} = \sigma_y \cdot W_{pl} = 250\ Mpa \times 1200000\ mm^3 = 300\ kN \cdot m$$



If the moment is 250 kN·m what is the plastic depth of the cross-section?





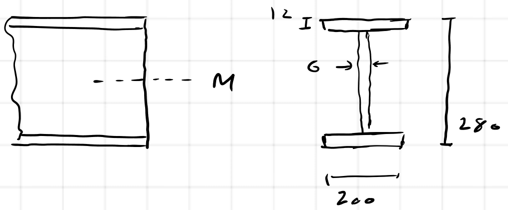
$$F_1 = \sigma \cdot A_1 = \overset{\sigma_y}{\sigma} \cdot (120 \cdot y)$$

$$F_2 = \frac{\sigma}{2} \cdot A_2 = \frac{\sigma_y}{2} ((100-y) \times 120)$$

$$M = F_1 \cdot (100 - \frac{y}{2}) \cdot 2 + F_2 \cdot \frac{2}{3} (100 - y) \cdot 2$$

$$\underbrace{250 \times 10^6}_{N \cdot mm} = \underbrace{250}_{Mpa} \cdot \underbrace{(120 \cdot y)}_{mm^2} \cdot \underbrace{(100 - \frac{y}{2})}_{mm} \cdot 2 + \underbrace{\frac{250}{2}}_{Mpa} \cdot \underbrace{((100-y) \times 120)}_{mm^2} \times \frac{2}{3} \underbrace{(100-y)}_{mm} \cdot 2$$

$\rightarrow y = 29.3\ mm$ plastic depth $\rightarrow 2 \times (29.3\ mm) = 58.6\ mm$



$W_{el} = 6.76 \times 10^5\ mm^3$

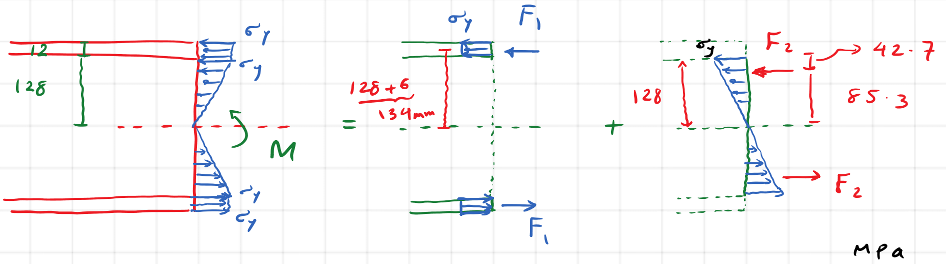
$W_{pl} = 7.41 \times 10^5\ mm^3$

$\sigma_y = 250\ Mpa$

$M_{el} = \sigma_y \cdot W_{el} = 250\ Mpa \times 6.76 \times 10^5\ mm^3 = 169\ kN \cdot m$

$M_{pl} = \sigma_y \cdot W_{pl} = 250\ Mpa \times 7.41 \times 10^5\ mm^3 = 185\ kN \cdot m$

what is the bending moment that the flange will be plastic?

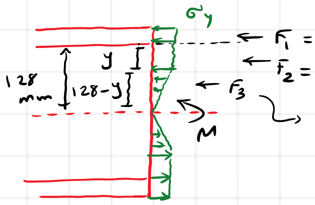


$F_1 = \sigma_y \cdot A_f = \underset{Mpa}{250} \times \underbrace{200 \times 12}_{mm^2} = \underset{kN}{600}$ $\qquad F_2 = \frac{\overset{Mpa}{250}}{2} \cdot \underbrace{128 \times 6}_{mm^2} = 96\ kN$

$M = F_1 \times 134 \times 2 + F_2 \times 85.3 \times 2 = 177.2\ kN \cdot m$

If $M = 180\ kN \cdot m$ what is the plastic depth of the cross-section?

$M = 180\ kN \cdot m > 177.2\ kN \cdot m \rightarrow$ partial part of the web is also plastic.



$F_1 = 600\ kN,\ \bar{y}_1 = 134\ mm$

$F_2 = y \times 6 \times \sigma_y,\ \bar{y}_2 = 128 - \frac{y}{2}$

F_3

$F_3 = (128-y) \times 6 \times \sigma_y / 2,\ \bar{y}_3 = \frac{2}{3}(128-y)$

$$M = 2\left[F_1 \cdot \bar{y}_1 + F_2 \cdot \bar{y}_2 + F_3 \cdot \bar{y}_3\right] \Rightarrow \underbrace{180 \times 10^6}_{N \cdot mm} = 2\left[\underbrace{600000}_{N} \times \underbrace{134}_{mm} + 250 \times 6 \times y \times \left(128 - \frac{y}{2}\right) + \frac{250}{2} \times 6(128-y) \cdot \frac{2}{3}(128-y)\right]$$

$\rightarrow y = 11.82\ mm$

Plastic depth $2(\underset{web}{11.82} + \underset{flange}{12}) = 47.64\ mm$