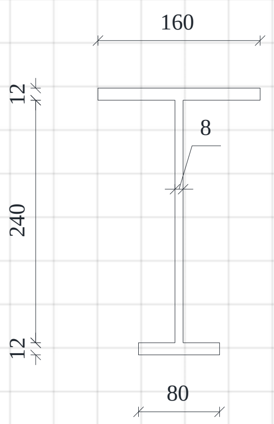
In the previous video, the elastic and plastic bending moment was determined.

a) Determine the bending moment that the compressive flange yields.

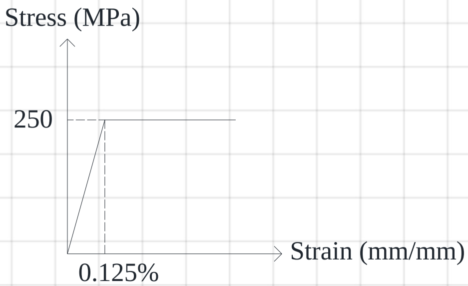
If the bending moment applied to the shown cross-section is 125kN.m:

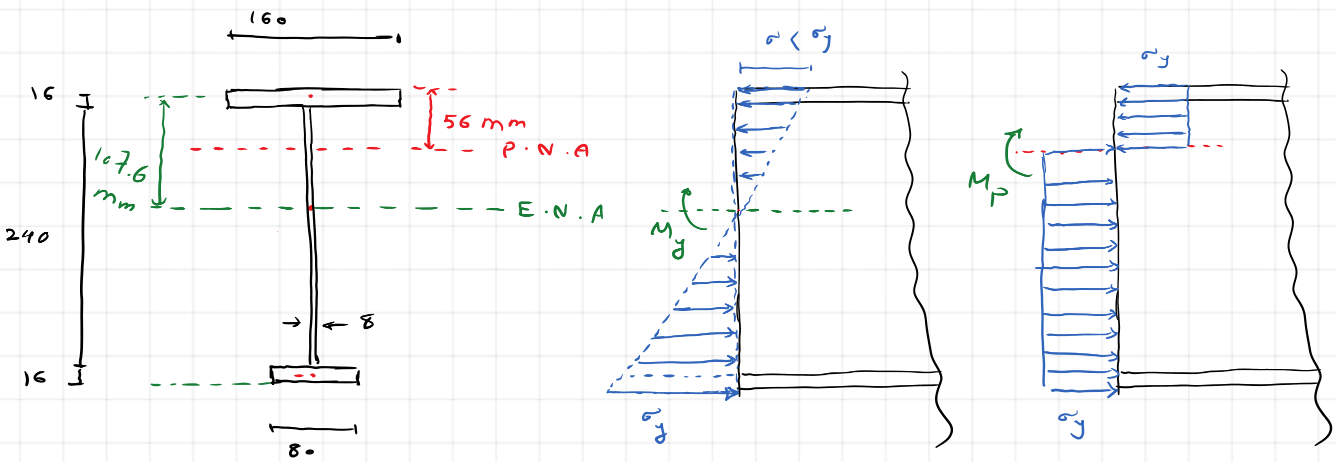
b) Determine the depth of the cross-section, which is elastic.
c) What is the maximum compressive stress on the cross-section.

P.S. Assume that the bending moment applied to the cross-section is positive.



Cross-Section (mm)

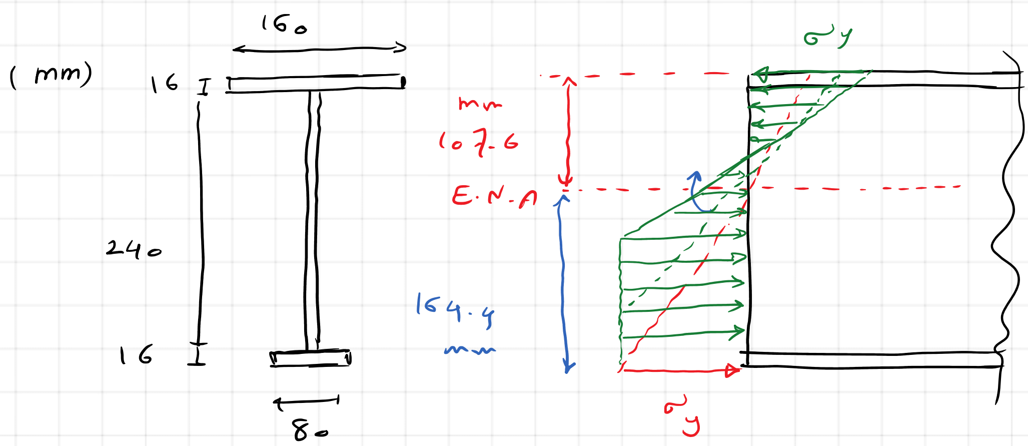




$$M_y = W_{el} \cdot \sigma_y = 4.1 \times 10^5\ mm^3 \times 250\ MPa \simeq 102\ kN \cdot m \qquad (\sigma_y = 250\ MPa)$$

$$M_p = W_{pl} \cdot \sigma_y = 5.55 \times 10^5\ mm^3 \times 250\ MPa = 137.5\ kN \cdot m$$

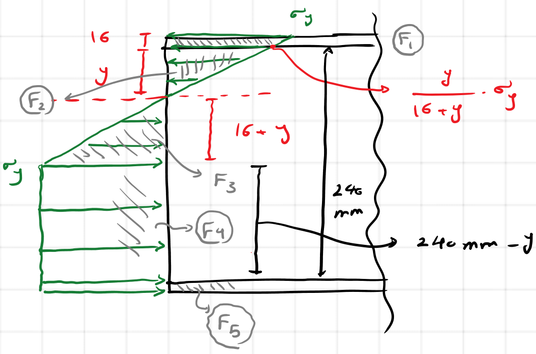
$$M = 125\ kN \cdot m \rightarrow \begin{array}{l} M > M_y \rightarrow \text{Not elastic} \\ M < M_p \rightarrow \text{Partially plastic} \end{array}$$



$$\sigma_{Tf} = \frac{102\ kN \cdot m \times 107.6\ mm}{6.75 \times 10^7\ mm^4} = 163\ MPa$$

$$M_y = 102\ kN \cdot m$$

$$\sigma_{Bf} = \frac{M \cdot y}{I} = 250\ MPa$$



$\leftarrow F_1$

$\leftarrow F_2$

$\rightarrow F_3$

$\rightarrow F_4$

$\rightarrow F_5$

$240\,mm - y - 16 - y = 240 - 16 - 2y = 224 - 2y$

Part 1 (Top flange), $A = 160 \times 16\ mm^2$, $\bar{\sigma} = \dfrac{\sigma_y + \frac{y}{16+y}\cdot\sigma_y}{2}$

Part 2 (web in compression), $A = y \times 8\ mm^2$, $\bar{\sigma} = \dfrac{y}{16+y}\cdot\sigma_y\cdot\dfrac{1}{2}$

Part 3 (web in tension), $A = (16+y)\times 8\ mm^2$, $\bar{\sigma} = \dfrac{\sigma_y}{2}$

Part 4 (web in plastic phase), $A = (224-2y)\times 8\ mm^2$, $\bar{\sigma} = \sigma_y$

Part 5 (Bottom Flange), $A = 80 \times 16\ mm^2$, $\bar{\sigma} = \sigma_y$

$F_1 = \bar{\sigma}_1 \cdot A_1$

$F_2 = \bar{\sigma}_2 \cdot A_2$

$F_3 = \bar{\sigma}_3 \cdot A_3$

$F_4 = \bar{\sigma}_4 \cdot A_4$

$F_5 = \bar{\sigma}_5 \cdot A_5$

$\sum F_x = 0 \Rightarrow \quad F_1 + F_2 = F_3 + F_4 + F_5$

$$\cancel{\sigma_y}\left[\frac{1+\frac{y}{16+y}}{2}\cdot 160\times 16 + \frac{y}{16+y}\cdot\frac{1}{2}\cdot y\cdot 8\right] = \cancel{\sigma_y}\left[\frac{1}{2}\cdot(16+y)\cdot 8 + 1\times(224-2y)\cdot 8 + 1\times 80\times 16\right]$$

$y = 56.72\ mm$

$\sigma_y = 250\ Mpa \rightarrow \bar{\sigma}_1 = \left.\dfrac{\sigma_y + \frac{y}{16+y}\cdot\sigma_y}{2}\right|_{y=56.72mm} \longrightarrow \bar{\sigma}_1 = 222.5\ Mpa,\ A_1 = 160\times 16\ mm^2$

$F_1 = \bar{\sigma}_1 \cdot A_1 = 570\ kN$

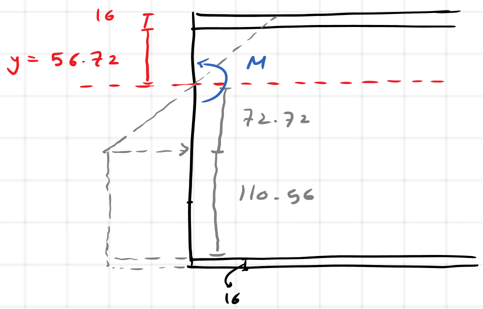
$\bar{\sigma}_2 = 97.5\ Mpa,\ A_2 = 8y = 454\ mm^2 \rightarrow F_2 = 44.24\ kN$

$\bar{\sigma}_3 = 125\ Mpa,\ A_3 = 582\ mm^2 \rightarrow F_3 = 72.72\ kN$

$\bar{\sigma}_4 = 250\ Mpa,\ A_4 = 884.5\ mm^2 \rightarrow F_4 = 221.\ kN$

$\bar{\sigma}_5 = 250\ Mpa,\ A_5 = 80\times 16 mm^2 \rightarrow F_5 = 320\ kN$

$F_1 + F_2 \cong 614\ kN \qquad F_3 + F_4 + F_5 \cong 614\ kN \qquad$ (ok)



$\leftarrow F_1 = 570\ kN$, $y_1 = 56.72 + \frac{16}{2} = 64.71\ mm$

$\leftarrow F_2 = 44.24\ kN$, $y_2 = \frac{2}{3} y = 37.8\ mm$

$\rightarrow F_3 = 72.72\ kN$, $y_3 = \frac{2}{3} \times 72.72 = 48.48\ mm$

$\rightarrow F_4 = 221\ kN$, $y_4 = \frac{110.56}{2} + 72.72 = 128\ mm$

$\rightarrow F_5 = 320\ kN$, $y_5 = \frac{16}{2} + 110.56 + 72.72 = 191.28\ mm$

$M = \sum F_i\, y_i = 131.6\ kN \cdot m$

$M_y = 102\ kN \cdot m$

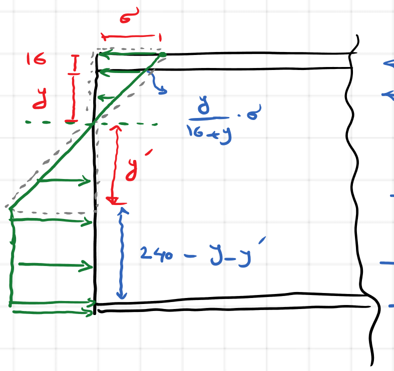
$M_p = 137.5\ kN \cdot m$

$M(\text{top flange yields}) = 131.6\ kN \cdot m$

$M = 125\ kN \cdot m$

(mm)

$\frac{\sigma}{16+y} = \frac{\sigma_y}{y'}$



$F_1 = \frac{\sigma + \frac{y}{16+y} \cdot \sigma}{2} \cdot 160 \times 16$, $y_1 = y + 8$

$F_2 = \frac{1}{2} \frac{y}{16+y} \cdot \sigma \cdot y \times 8$, $y_2 = \frac{2}{3} y$

$F_3 = \frac{1}{2} \cdot \sigma_y \cdot y' \cdot 8$, $y_3 = \frac{2}{3} y'$

$F_4 = \sigma_y \cdot (240 - y - y') \cdot 8$, $y_4 = y' + \frac{240 - y - y'}{2}$

$F_5 = \sigma_y \cdot 80 \times 16$, $y_5 = y' + 240 - y - y' + 8$

$\sum F = 0 \Rightarrow F_1 + F_2 = F_3 + F_4 + F_5$

$\sum M = 125\ kN \cdot m \rightarrow F_1 \cdot y_1 + F_2\, y_2 + \cdots + F_5 \cdot y_5 = 125\ kN \cdot m$

Guess Values

$y := 60\ mm \qquad \sigma := 200\ MPa$

Constraints

$F_1(\sigma, y) + F_2(\sigma, y) = F_3(\sigma, y) + F_4(\sigma, y) + F_5(\sigma, y)$

$M = F_1(\sigma, y) \cdot y_1(\sigma, y) + F_2(\sigma, y) \cdot y_2(\sigma, y) + F_3(\sigma, y) \cdot y_3(\sigma, y) + F_4(\sigma, y) \cdot y_4(\sigma, y) + F_5(\sigma, y) \cdot y_5(\sigma, y)$

Solver

$\begin{bmatrix} \sigma \\ y \end{bmatrix} := \text{Find}(\sigma, y) = \begin{bmatrix} (2.206 \cdot 10^8)\ Pa \\ 0.07\ m \end{bmatrix}$

$y' = 97.4\ mm$

depth of the section in elastic $\rightarrow 16 + y + y' = 183.4\ mm$

$\sigma = 220.636\ MPa$

$\sigma_{\text{on the top flange}} = 220\ MPa$

$y = 69.997\ mm$