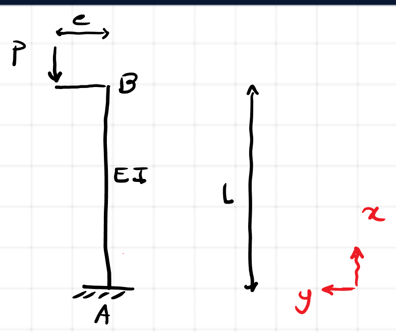
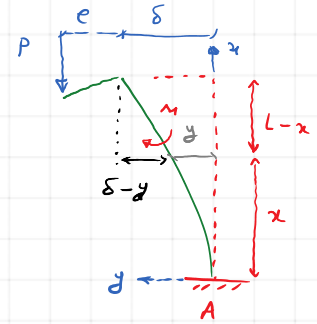
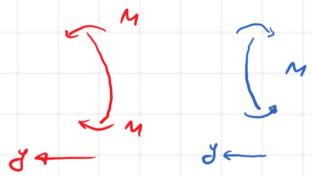
In a cantilever column, the compressive load is applied by the eccentricity of e, as shown in the figure. The column height is l with the bending rigidity of EI.

a) Derive the maximum horizontal displacement of point B as a function of eccentricity, force p, and EI.
b) Sketch the relation between the maximum displacement and the applied force p[1].
c) Determine the maximum bending moment and maximum normal stress.

[1] The best solution would be using the non-dimensional variables







$M = E \cdot I \cdot y''$ $M = - E \cdot I \cdot y''$

$$M = P(e+\delta-y) = P(e+\delta) - P\cdot y$$

$$E\cdot I \cdot y'' = P(e+\delta) - Py$$

$$EI\cdot y'' + P\cdot y = P\cdot(e+\delta)$$

$$y'' + \frac{P}{EI}\cdot y = \frac{P}{E\cdot I}(e+\delta) \qquad \frac{P}{EI} = \lambda^2$$

$$y'' + \lambda^2 y = \lambda^2(e+\delta) \Rightarrow$$

$$y = A\cdot \sin\lambda x + B\cos\lambda x + (e+\delta) \;,\; y' = A\cdot\lambda\cdot\cos\lambda x - B\cdot\lambda\cdot\sin\lambda x$$

$$\begin{cases} y(0)=0 & \rightarrow \quad B+e+\delta = 0 \Rightarrow B = -e-\delta \\ y'(0)=0 & \rightarrow \quad A\cdot\lambda = 0 \quad \Rightarrow \quad A = 0 \end{cases}$$

$$y = -(e+\delta)\cos\lambda x + (e+\delta) = (e+\delta)\{-\cos\lambda x + 1\}$$

$$y = (e+\delta)\left[1-\cos\lambda x\right]$$

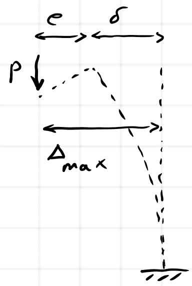
$$y(L)=\delta \Rightarrow \quad \delta = (e+\delta)\left[1-\cos\lambda L\right]$$

$$\cancel{\delta} = e + \cancel{\delta} - (e+\delta)\cos\lambda L \Rightarrow e - e\cos\lambda L - \delta\cdot\cos\lambda L = 0 \Rightarrow$$

$$\delta\cos\lambda L = e - e\cos\lambda L \Rightarrow \delta = e\left(\frac{1-\cos\lambda L}{\cos\lambda L}\right) = e\left(\frac{1}{\cos\lambda L} - 1\right)$$

$$\rightarrow \boxed{\delta = e(\sec\lambda L - 1)}$$

$$\delta = e\,(\underbrace{\sec\lambda L}_{} - 1)$$



$$\Delta_{max} = \delta + e = e\,\sec\lambda L - e + e$$

$$\boxed{\Delta_{max} = e \cdot \sec\lambda L}$$

$$\boxed{e = \alpha \cdot L}$$

$$\lambda^2 = \frac{P}{EI} \Rightarrow \lambda^2 \cdot L^2 = \frac{PL^2}{EI} = \frac{P}{EI/L^2} = \frac{P}{P_E \cdot \frac{4}{\pi^2}} = \frac{\pi^2}{4}\,\frac{P}{P_E}$$

$$r^2$$

$$P_E = \frac{\pi^2 EI}{(2L)^2}$$

$$P_E = \frac{\pi^2 EI}{4 \cdot L^2}$$

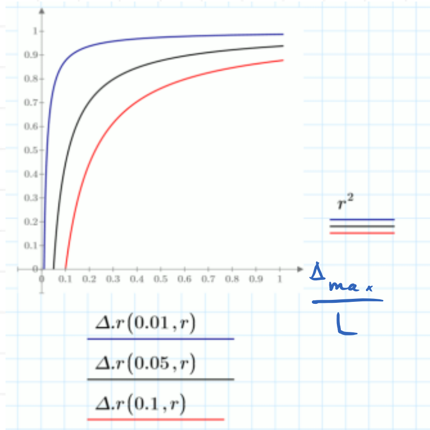
$$P_E \cdot \frac{4}{\pi^2} = \frac{EI}{L^2}$$

$$\lambda^2 \cdot L^2 = \frac{\pi^2}{4} \cdot r^2 \Rightarrow \boxed{\lambda L = \frac{\pi}{2} \cdot r}$$

$$\Delta_{max} = \alpha \cdot L \cdot \sec\left(\frac{\pi}{2} \cdot r\right)$$

$$\frac{\Delta_{max}}{L} = \alpha \cdot \sec\left(\frac{\pi}{2} \cdot r\right)$$

$$r^2 = \frac{P}{P_E}$$

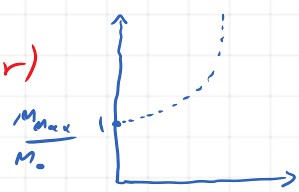


$$M_{max} = P \cdot \Delta_{max} = P \cdot \underbrace{\alpha \cdot L}_{e} \cdot \sec\left(\frac{\pi}{2} \cdot r\right)$$

$$M_{max} = P \cdot e \cdot \sec\left(\frac{\pi}{2}\, r\right)$$

$$\frac{M_{max}}{P \cdot e} = \sec\left(\frac{\pi}{2} \cdot r\right)$$

$$M_0 = P \cdot e$$



$$\sigma = \frac{P}{A} + \frac{M}{W}$$

$$\sigma_{max} = \frac{P}{A} + \frac{P \cdot e \cdot \sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_E}}\right)}{W}$$

$$\sigma_{max} = P\left[\frac{1}{A} + \frac{e \cdot \sec\left(\frac{\pi}{2}\sqrt{P/P_E}\right)}{W = \frac{I}{C}}\right]$$