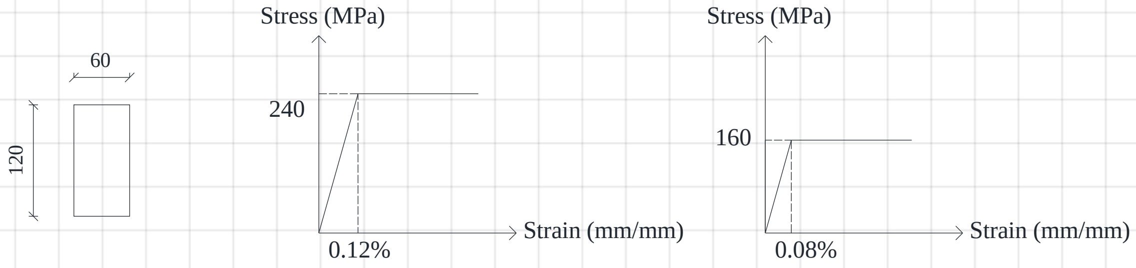
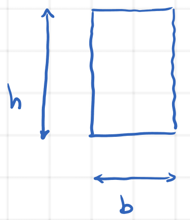
A rectangular cross-section is made of an elastic, perfectly plastic material behaving differently in tension and compression. However, the material's elastic modulus in tension and compression is assumed to be the same. The relation between stress and strain for the tension and compression is shown below.

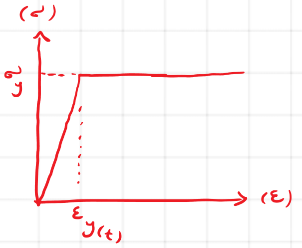
a) Determine the bending moment the cross-section approaches its yield limit.
b) What is the elastic neutral axis?
c) Which side of the cross-section will yield first, the tension or the compression side?
d) As noticed, the compressive side of the cross-section will yield first. Determine the required bending moment that the tensile side of the cross-section yields. Also, determine the location of the neutral axis in this condition.
e) If the cross-section is entirely plastic, determine the plastic neutral axis and the corresponding plastic bending moment.

P.S. Assume that the bending moment applied to the cross-section is positive.

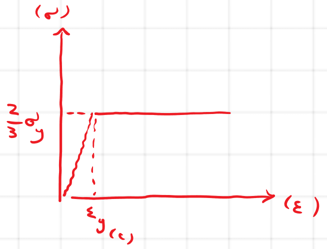


Rectangular Cross-Section (mm) Tensile behavior Compressive behavior





Tension



Compression

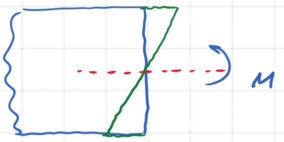
$(\varepsilon_c = \varepsilon_t)$

$\sigma_y = 240\ MPa$

$E = 200\ GPa$

$\sigma_{y(t)} = 240\ MPa$

$\sigma_{y(c)} = 160\ MPa$



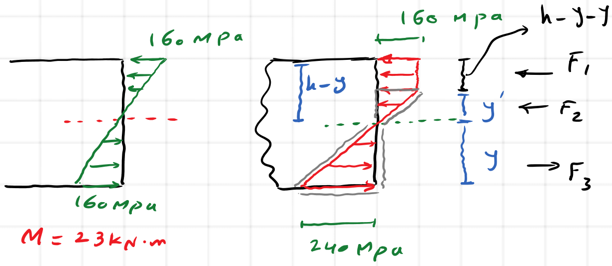
$$\left(I = \frac{bh^3}{12}\right)$$

$$W_{el} = \frac{I}{c} = \frac{bh^2}{6}$$

$$\sigma_{max(c)} = \frac{M \cdot c_c}{I} = \frac{M}{W_{el}} \Rightarrow$$

$$160\ MPa = \frac{M_{el}}{\frac{bh^2}{6}} \Bigg\} \Rightarrow \boxed{M_{el} = 23\ kN.m}$$

$$\begin{cases} b = 60\ mm \\ h = 120\ mm \end{cases} \rightarrow W_{el} = 1.44 \times 10^5\ mm^3$$



$$h - y - y' = h - y - \frac{2}{3}y = h - \frac{5}{3}y$$

$$\frac{160}{y'} = \frac{240}{y} \rightarrow \boxed{y' = \frac{2}{3}y}$$

$$F_1 = 160\ MPa \times b \times \left(h - \frac{5}{3}y\right)$$

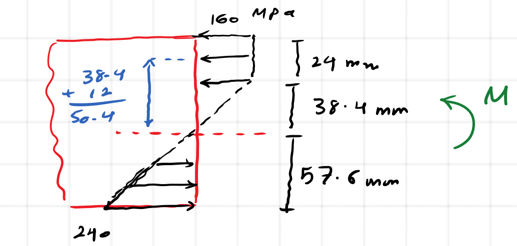
$$F_2 = \frac{1}{2} \times 160\ MPa \times b \times y' = \frac{160\ MPa}{2} \cdot b \cdot \frac{2}{3}y \qquad F_1 + F_2 = F_3$$

$$F_3 = \frac{1}{2} \times 240\ MPa \times b \cdot y$$

$$\rightarrow \cancel{160}^{2}\ MPa \times \cancel{b}\left(h - \frac{5}{3}y\right) + \frac{\cancel{160}^{2}\ MPa}{2} \cdot \cancel{b} \cdot \frac{2}{3}y = \frac{\cancel{240}^{3}\ MPa}{2} \cdot \cancel{b} \cdot y$$

$$6\left(2h - \frac{10}{3}y + \frac{2}{3}y = \frac{3}{2}y\right)$$

$$12h - 20y + 4y = 9y \Rightarrow 12h = 25y \rightarrow \boxed{y = 0.48h} \rightarrow y = 57.6\ mm$$

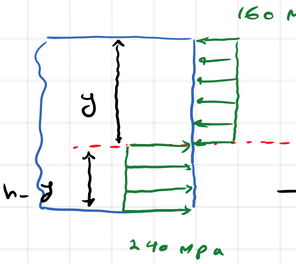


$\leftarrow F_1 = \underset{MPa}{160} \cdot \underset{mm}{24} \cdot \underset{mm}{60} = \underset{kN}{230.4}$ $\qquad y_1 = 50.4\,mm$

$\leftarrow F_2 = \frac{160\,MPa}{2} \cdot \underset{mm}{38.4} \times \underset{mm}{60} = \underset{kN}{184.32}$ $\qquad y_2 = \frac{2}{3} \times \underset{mm}{38.4} = 25.6\,mm$

$\rightarrow F_3 = \frac{240\,MPa}{2} \times \underset{mm}{57.6} \times \underset{mm}{60} = \underset{kN}{414.72}$ $\qquad y_3 = \frac{2}{3} \times 57.6 = 38.4\,mm$

$$M = \sum F_i \cdot y_i = F_1 y_1 + F_2 y_2 + F_3 y_3 = 32.26\ kN \cdot m$$



$\leftarrow F_1 = 160\,MPa \times y \times b$

$\rightarrow F_2 = 240\,MPa \times (h-y) \times b$

$F_1 = F_2 \Rightarrow$

$\cancel{160}^{2}\,MPa \cdot y \cdot \cancel{b} = \cancel{240}^{3}\,MPa \cdot (h-y) \cdot \cancel{b}$

$\rightarrow 2y = 3h - 3y \Rightarrow 5y = 3h \rightarrow \boxed{y = 0.6\,h}$ $\Big|_{h = 120\,mm} \rightarrow y = 72\,mm$

$F_1 = 160\,MPa \times 72\,mm \times 60\,mm = 691.2\ kN$ $\qquad y_1 = y/2 = 36\,mm$

$F_2 = 240\,MPa \times (120\,mm - 72\,mm) \times 60\,mm = 691.2\,kN$ $\qquad y_2 = \frac{h-y}{2} = 24\,mm$

$$M_p = \sum F_i \cdot y_i = F_1 \cdot y_1 + F_2 \cdot y_2 = 41.47\ kN \cdot m$$

$M < M_{el} \Rightarrow$ in tension & com the cross-section is elastic

$M_{el} = 23\ kN \cdot m$

$M^* = 32.26\ kN \cdot m$ $\qquad M_{el} < M < M^* \rightarrow$ The compression side is partially plastic, in the tensile side the cross-section is elastic

$M_{pl} = 41.47\,kN \cdot m$

$M^* < M < M_{pl} \rightarrow$ partially plastic in tension & compression side