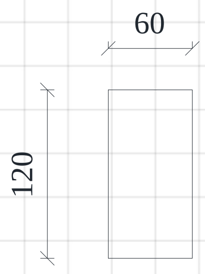
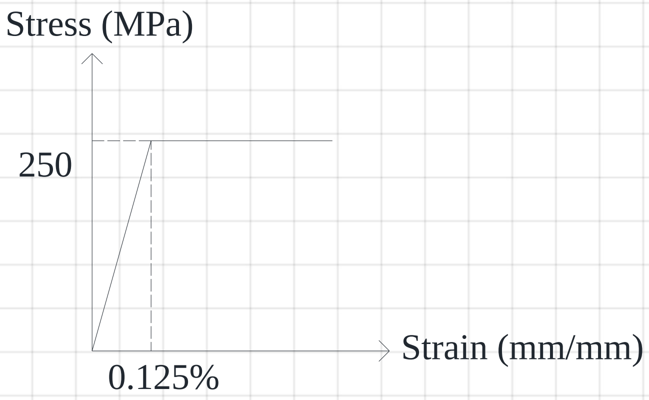
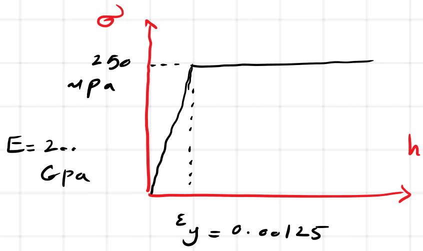
The shown rectangular cross-section is made of an elastic, perfectly plastic material.

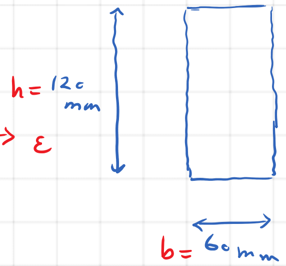
a) Determine the elastic and plastic bending moment.
b) If the cross-section is under a bending moment of 50kN.m, what is the depth of the elastic section?
c) If the applied bending moment is removed from the cross-section, determine the residual stress on the cross-section.
d) Derive the relation between the bending moment and the maximum strain of the cross-section on its edge.
e) Sketch the function of bending moment with the dependent variable of the furthest edge strain. Clearly show the linear and non-linear behavior of the cross-section.
f) Determine the permanent strain of the furthest cross-section edge considering two methods. First, with the derived graph from part e. Second, with the stress-strain diagram.
g) Cross-check your calculation with the permanent strain.





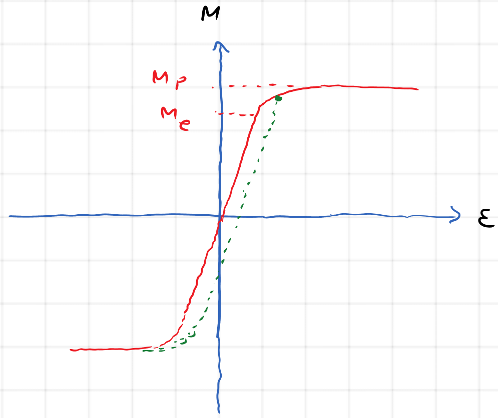
Rectangular Cross-Section (mm)



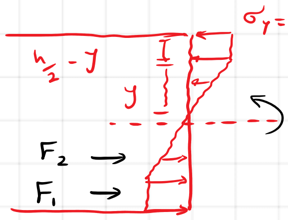


$$W_{el} = \frac{bh^2}{6} = 144000 \text{ mm}^3 \qquad M_{el} = \sigma_y \cdot W_{el} = 36 \text{ kN}\cdot\text{m}$$

$$W_{pl} = \frac{bh^2}{4} = 216000 \text{ mm}^3 \qquad M_{pl} = \sigma_y \cdot W_{pl} = 54 \text{ kN}\cdot\text{m}$$



$M_{el} < M = 50 \text{ kN}\cdot\text{m} < M_{pl} \rightarrow$ partially plastic



$$F_1 = \sigma_y \cdot b \cdot \left(\frac{h}{2} - y\right), \quad y_1 = y + \frac{h/2 - y}{2} = \frac{h/2 + y}{2}$$

$$F_2 = \frac{\sigma_y}{2} \cdot b \cdot y, \quad y_2 = \frac{2}{3} y$$

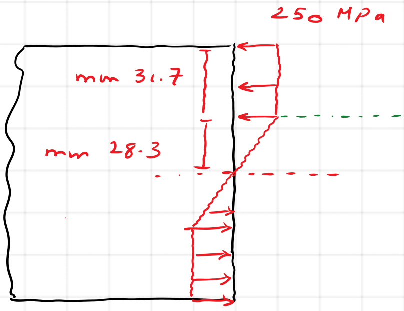
$$M = \sum F_i \cdot y_i = 2\left[F_1 y_1 + F_2 y_2\right] \Rightarrow$$

$$50 \text{ kN}\cdot\text{m} = 2\left[\sigma_y \cdot b \cdot \left(\frac{h}{2} - y\right) \cdot \left(\frac{h/2 + y}{2}\right) + \frac{\sigma_y}{2} \cdot b \cdot y \cdot \frac{2}{3} y\right]$$

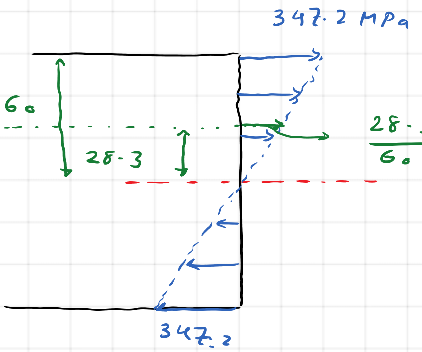
250 MPa, 60 mm, 120 mm

$\rightarrow y = 28.3 \text{ mm}$

unloading: $\sigma = \frac{M \cdot y}{I} = \frac{M}{W_{el}} \rightarrow \sigma = \frac{50 \times 10^6 \text{ N}\cdot\text{mm}}{144000 \text{ mm}^3} = 347.2 \text{ MPa} < \underbrace{\sigma_{y(t)} + \sigma_{y(c)}}_{500 \text{ MPa}}$ ✓



loading

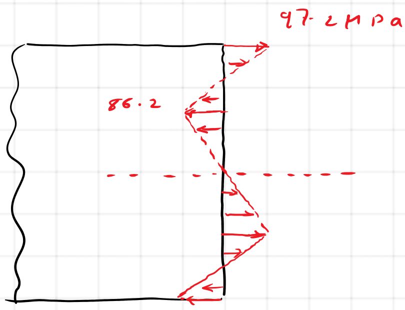


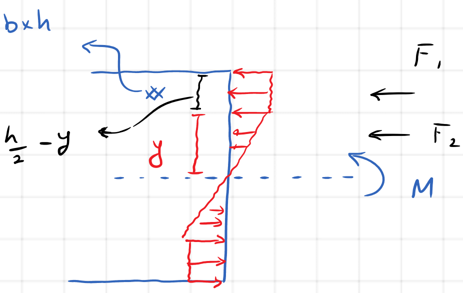
$$\frac{28.3}{60} \times 347.2 = 163.8\ MPa$$

unloading

$$-250\ MPa + 347.2\ MPa = 97.2\ MPa$$

$$-250\ MPa + 163.8\ MPa = -86.2\ MPa$$



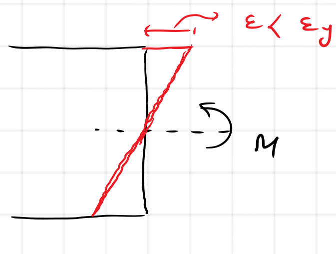


$$M_{el} = \sigma_y \cdot \frac{bh^2}{6} \quad , \quad M_{pl} = \sigma_y \cdot \frac{bh^2}{4}$$

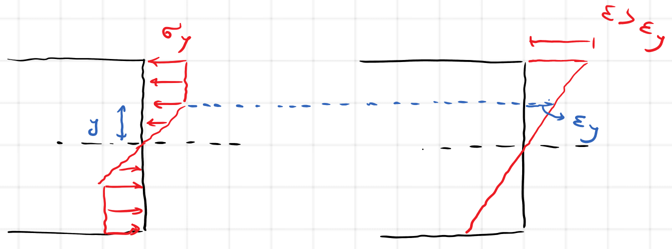
$$F_1 = b \cdot \left(\frac{h}{2} - y\right) \cdot \sigma_y \qquad y_1 = y + \left(\frac{h}{2} - y\right) \div 2 = y + \frac{h/2 - y}{2} = \frac{h/2 + y}{2}$$

$$F_2 = b \cdot y \cdot \frac{\sigma_y}{2} \qquad y_2 = \frac{2}{3} \cdot y$$

$$M = \Sigma F_i y_i = 2\left\{F_1 \cdot y_1 + F_2\, y_2\right\} = 2\left[\sigma_y \cdot b\left(\frac{h}{2} - y\right)\left(\frac{h/2 + y}{2}\right) + \sigma_y \cdot \frac{1}{2} \cdot b \cdot y \cdot \frac{2}{3}\, y\right]$$



(Strain)



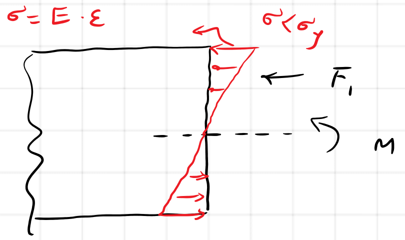
(Stress) (Strain)

$M < M_{el}$

$M_{el} < M < M_{pl}$

$\sigma = E \cdot \varepsilon$

$$\frac{\varepsilon}{h/2} = \frac{\varepsilon_y}{y} \Rightarrow \boxed{y = \frac{h}{2} \cdot \frac{\varepsilon_y}{\varepsilon}} \quad \text{I}$$



(Stress)

$(M < M_{el})$

$$M = F_1 \cdot y_1 \cdot 2 = \frac{\sigma}{2} \cdot \frac{h}{\cancel{2}} \cdot b \cdot \frac{\cancel{2}}{3} \cdot \frac{h}{\cancel{2}} \cdot \cancel{2} = \overset{E \cdot \varepsilon}{\sigma} \cdot \frac{bh^2}{6}$$

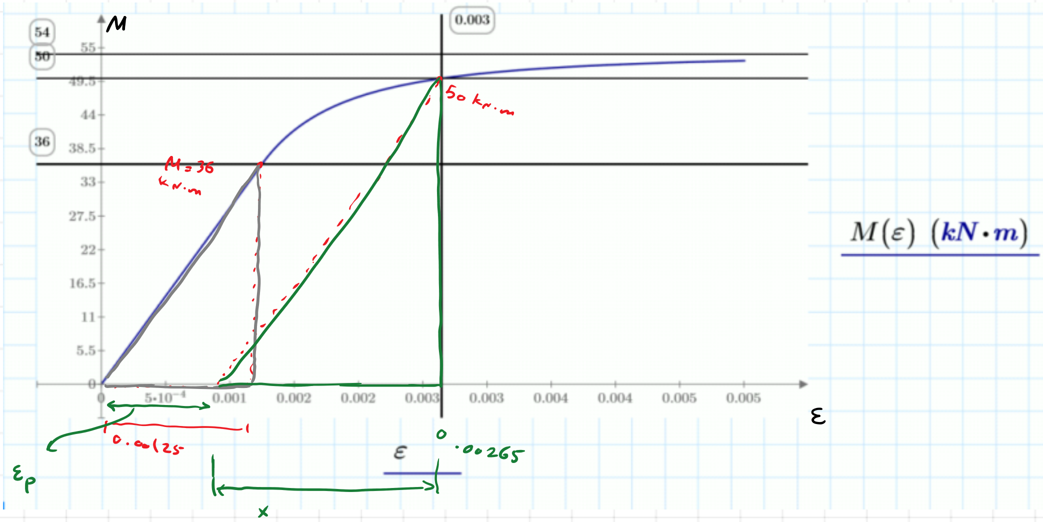
$$M = E \cdot \varepsilon \cdot \frac{bh^2}{6}$$

$M < M_{el}$

$M_{el} < M < M_{pl}$

$$M(b,h,\sigma_y,y) := 2 \cdot \sigma_y \cdot b \cdot \left(\frac{1}{2} \cdot \left(\frac{h}{2} - y\right) \cdot \left(\frac{h}{2} + y\right) + \frac{1}{3} \cdot y^2\right)$$

I $\quad y = \frac{h}{2} \cdot \frac{\varepsilon_y}{\varepsilon}$

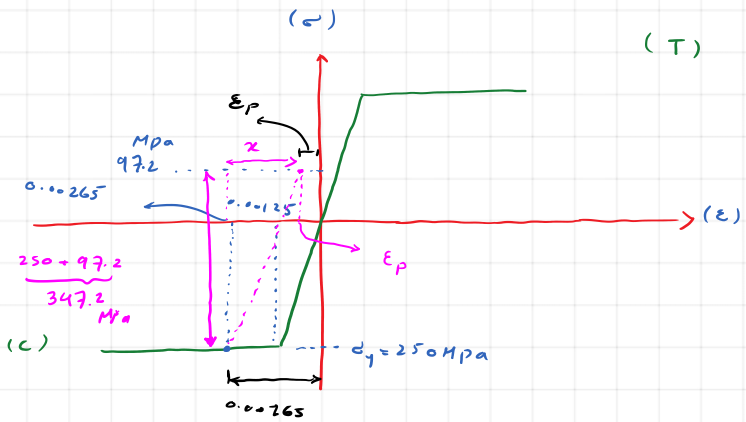


$$\frac{50}{x} = \frac{36}{0.00125}$$

$$x = 0.00174$$

$$\varepsilon_p = 0.00265 - 0.00174$$

$$\boxed{\varepsilon_p = 0.00091}$$



$$\frac{347.2}{x} = \frac{250}{0.00125}$$

$$x = 0.00174$$

$$\varepsilon_p = 0.00265 - x = \boxed{0.00091}$$