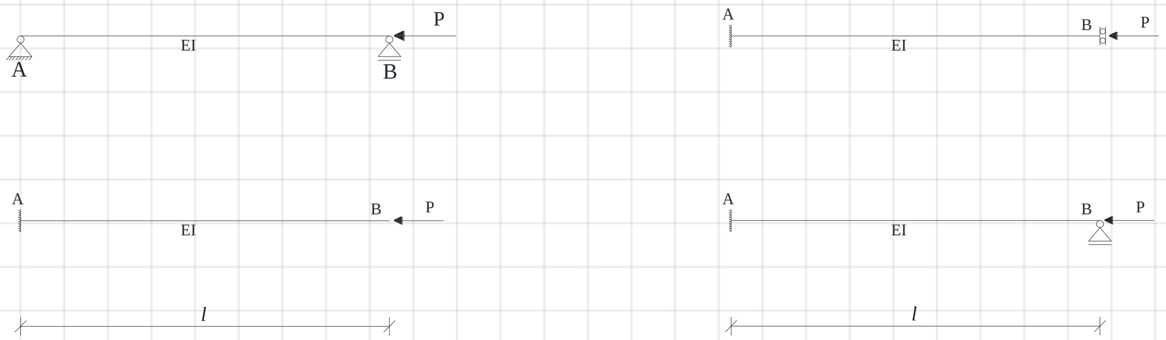
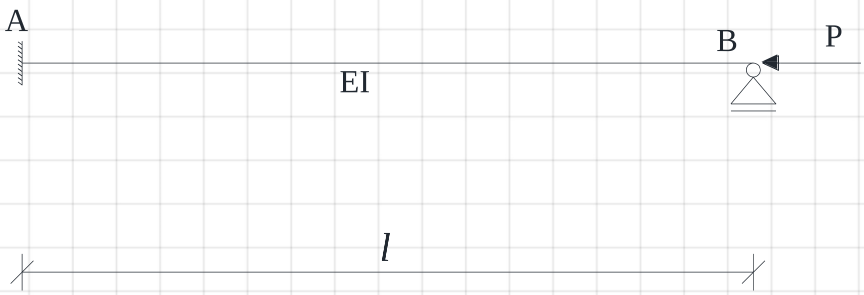
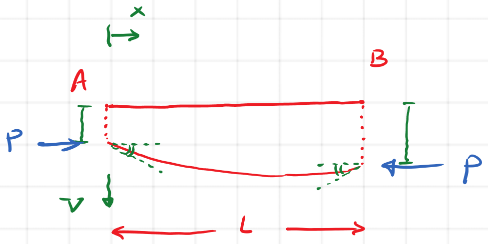
In the stability analysis there are different definitions for boundary conditions. Essential boundary conditions also known as kinematic boundary conditions, neutral boundary conditions and equilibrium boundary conditions. In the general form, determine the general essential, neutral and equilibrium boundary conditions.

Then, write each boundary condition for different beam conditions based on their categories.



For the beam shown below, determine the first buckling load.





$$\Pi = \frac{1}{2}\int_0^L EI\,k^2\,dx - \frac{P}{2}\int_0^L v'^2\,dx$$

$$k = -\frac{v''}{\sqrt{1-v'^2}} = -v''\cdot\left(1+\underbrace{\tfrac{1}{2}v'^2}_{\to 0}\right) \cong -v''$$

$$\Pi = \frac{1}{2}\int_0^L EI\,v''^2\,dx - \frac{P}{2}\int_0^L v'^2\,dx$$

$$\delta\Pi = \frac{1}{\cancel{2}}\int_0^L E\cdot I\cdot \cancel{2}v''\cdot\delta(v'')\,dx - \underbrace{\frac{P}{\cancel{2}}\int_0^L \cancel{2}v'\,\delta v'\,dx}_{③}$$

$$\int_0^L \underbrace{E\cdot I\cdot v''}_{u}\cdot\underbrace{\delta(v'')}_{d\bar{v}}\cdot dx \Rightarrow ①$$

$$\boxed{\int u\cdot dv = u\cdot\bar{v} - \int \bar{v}\cdot du}$$

$$du = (EI\,v'')' \qquad \bar{v} = \delta v'$$

$$① = EI\,v''\cdot\delta v' - \overbrace{\int \underbrace{(EI\,v'')'}_{u}\cdot\underbrace{\delta v'}_{d\bar{v}}\cdot dx}^{②}$$

$$du = (EI\,v'')'' \;,\; \bar{v} = \delta v$$

$$② = (EI\,v'')'\cdot\delta v - \int (EI\,v'')''\cdot\delta v\cdot dx$$

$$③ \Rightarrow \quad P \int_0^L \underbrace{v'}_{u} \cdot \underbrace{\delta v'}_{d\bar{v}} \cdot dx$$

$$du = (v')' , \quad \bar{v} = \delta v$$

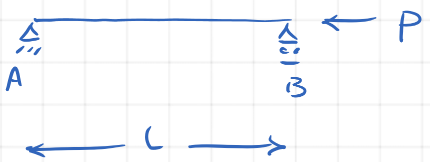
$$③ = P\left[v' \cdot \delta \bar{v} - \int_0^L (v')' \cdot \delta \bar{v} \cdot dx \right]$$

$$\delta \Pi = \Big| _0^L EI\, v'' \cdot \delta v' - \Big| _0^L (EI\, v'')' \cdot \delta v + \int_0^L (EI\, v'')'' \cdot \delta v \cdot dx$$

$$- \Big| _0^L P \cdot v' \cdot \delta v + \int_0^L P(v')' \cdot \delta \bar{v} \cdot dx$$

$$\delta \Pi = \Big| _0^L EI\, v'' \cdot \delta v' - \Big| _0^L \left((EI\, v'')' + P v' \right) \delta v + \int_0^L \left((EI\, v'')'' + (P v')' \right) \delta v \cdot dx$$

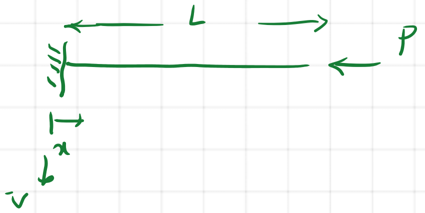
$$\begin{matrix} EI\, v''(L) \cdot \delta v'(L) \\ - EI\, v''(0) \cdot \delta v'(0) \end{matrix} - \left[\begin{matrix} \left((EI\, v'')'(L) + P v'(L) \right) \delta v(L) \\ - \left((EI\, v'')'(0) + P v'(0) \right) \delta v(0) \end{matrix} \right] + \int \left(EI\, v^{(4)} + P v'' \right) \delta \bar{v} \cdot dx \quad \mathrm{I}$$



$$\begin{cases} EI\, v'' = -M \\ (EI\, v'')' = -Q \end{cases}$$

$$\mathrm{I} \quad \begin{matrix} -\cancel{M(L) \cdot \delta v'(L)} \\ +\cancel{M(0) \cdot \delta v'(0)} \end{matrix} - \left[\begin{matrix} \left(-Q(L) + P v'(L) \right) \delta v(L) \\ - \left(-Q(0) + P v'(0) \right) \cdot \delta v(0) \end{matrix} \right] + \int \left(EI\, v^{4} + P v'' \right) \delta v \cdot dx$$

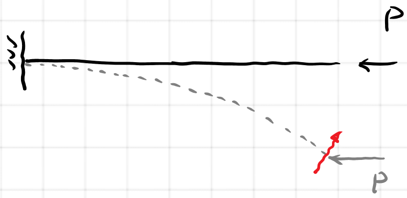
$$\underbrace{\begin{cases} M(L) = 0 \\ M(0) = 0 \end{cases}}_{\text{Neutral B.C}} \qquad \underbrace{\begin{cases} \delta v(L) = 0 \\ \delta v(0) = 0 \end{cases}}_{\text{essential B.C}} \qquad \underbrace{EI\, v^{(4)} + P v'' = 0}_{\text{Equilibrium B.C}}$$



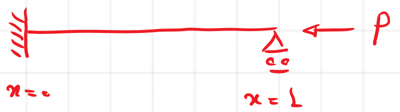
$$\text{I} \quad -M(L)\,\delta v'(L) + \cancel{M(0)\,\delta v'(0)} - \left[\left(-Q(L) + P\, v'(L)\right)\delta v(L) - \cancel{\left(-Q(0) + P v'(0)\right)\delta v(0)} \right] + \int \left(EI\, v^{4} + P v''\right)\delta v \, dx$$

$x = 0 \Rightarrow \delta v(0) = 0$
$x = 0 \Rightarrow \delta v'(0) = 0$ } Essential B.C

$\left\{ \begin{array}{l} M(L) = 0 \\ Q(L) - P v'(L) = 0 \end{array} \right.$ } Neutral B.C



$EI\, v^{(4)} + P v'' = 0$ → Equilibrium B.C

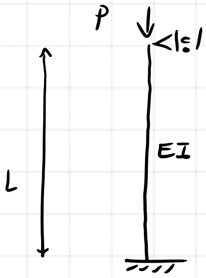


$$\text{I} \quad -M(L)\,\delta v'(L) + \cancel{M(0)\,\delta v'(0)} - \left[\cancel{\left(-Q(L) + P\, v'(L)\right)\delta v(L)} - \left(\cancel{-Q(0)} + P v'(0)\right)\delta v(0) \right] + \int \left(EI\, v^{4} + P v''\right)\delta v \, dx$$

$\delta v(0) = 0$ $\qquad \delta v'(0) = 0$ $\qquad \delta v(L) = 0$ $\qquad \delta v'(L) \neq 0 \Rightarrow M(L) = 0$

$$\text{I} \quad \cancel{-M(L)\,\delta v'(L)} + \cancel{M(0)\,\delta v'(0)} - \left[\left(-Q(L) + P\, v'(L)\right)\delta v(L) - \cancel{\left(-Q(0) + P v'(0)\right)\delta v(0)} \right] + \int \left(EI\, v^{4} + P v''\right)\delta v \, dx$$

$\delta v(0) = 0$ $\qquad \delta v'(0) = 0$ $\qquad \delta v'(L) = 0$ $\qquad \delta v(L) \neq 0 \Rightarrow -Q(L) + P v'(L) = 0$



$$EI\,v^{(4)} + P\,v'' = 0$$

$v(0) = 0$

$v'(0) = 0$

I_x $v(L) = 0$, $M(L) = 0 \Rightarrow EI\,v''(L) = 0$

$$v^{(4)} + \frac{P}{EI}v'' = 0 \qquad \frac{P}{EI} = \lambda^2$$

$$v^{(4)} + \lambda^2 v'' = 0 \Rightarrow r^4 + \lambda^2 r^2 = 0 \Rightarrow$$

$$r^2\left[r^2 + \lambda^2\right] = 0 \rightarrow \begin{cases} r = 0 \\ r = 0 \\ r = \pm\lambda i \end{cases}$$

$$v_h = A\sin\lambda x + B\cos\lambda x + \cancel{e^{0x}}(Cx + D)$$

1

$$\boxed{v = A\sin\lambda x + B\cos\lambda x + Cx + D}$$

$$v' = A\lambda\cos\lambda x - B\lambda\sin\lambda x + C$$

$$v'' = -A\lambda^2\sin\lambda x - B\lambda^2\cos\lambda x$$

$v(0) = 0 \Rightarrow B + D = 0$

$v'(0) = 0 \Rightarrow A\lambda + C = 0$

$v(L) = 0 \Rightarrow A\sin\lambda L + B\cos\lambda L + CL + D = 0$

$v''(L) = 0 \Rightarrow +A\cancel{\lambda^2}\sin\lambda L + B\cancel{\lambda^2}\cos\lambda L = 0$

$$\begin{array}{cccc} A & B & C & D \end{array}$$
$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ \lambda & 0 & 1 & 0 \\ \sin\lambda L & \cos\lambda L & L & 1 \\ \sin\lambda L & \cos\lambda L & 0 & 0 \end{bmatrix} = T$$

$\det(T) = 0$

$$T(l,\lambda) := \begin{bmatrix} 0 & 1 & 0 & 1 \\ \lambda & 0 & 1 & 0 \\ \sin(\lambda\cdot l) & \cos(\lambda\cdot l) & l & 1 \\ \sin(\lambda\cdot l) & \cos(\lambda\cdot l) & 0 & 0 \end{bmatrix}$$

$$\det(T(l,\lambda)) \rightarrow -\sin(\lambda\cdot l) + \lambda\cdot l\cdot\cos(\lambda\cdot l)$$

$\lambda L\cos\lambda L - \sin\lambda L = 0$

$$\lambda \cdot L\cos\lambda L - \sin\lambda L = 0 \Rightarrow \lambda L = \tan\lambda L$$

$$\rightarrow \boxed{\lambda L - \tan\lambda L = 0}$$

$$(\lambda L)_1 = 4.493 \Rightarrow \lambda^2 L^2 = 4.493^2 = 20.187$$

$$(\lambda L)_2 = 7.725$$

$$\frac{P_{cr_1}}{EI} \cdot L^2 = 20.187 \Rightarrow P_{cr_1} = 20.187 \frac{EI}{L^2}$$

$$P_{cr_1} = \frac{\pi^2 \cdot EI}{L^2} \cdot \frac{20.187}{\pi^2} = \frac{\pi^2 EI}{L^2 \cdot \underbrace{\pi^2}_{20.187}} = \frac{\pi^2 EI}{0.49 L^2} = \frac{\pi^2 EI}{(0.7L)^2}$$

$$\boxed{L_{eff} = 0.7L}$$