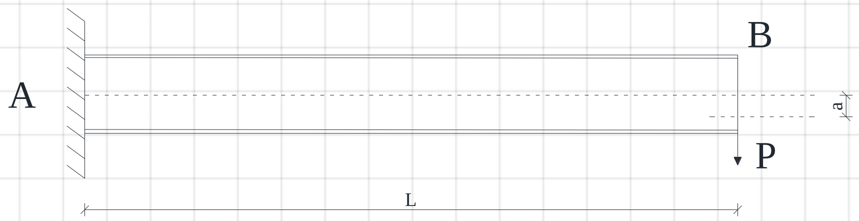
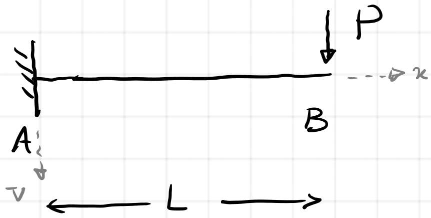
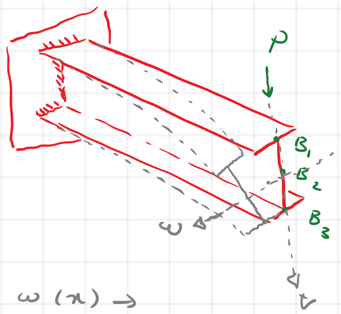
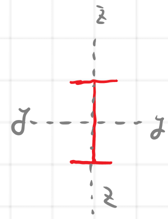
A cantilever I-Beam is subjected to a transversal load at its tip point B as shown in the figure. When the load P is increased, the beam starts to buckle laterally which is known as lateral torsional buckling.

a) Write the total potential energy equation considering the out-of-plane bending, torsion, and warping of the beam.
b) Take the deformation functions as reasonable admissible functions.
c) Determine the buckling load based on the location of force P.
d) Compare the results with the available sources like stability books and the Timoshenko equation.









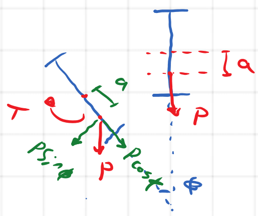
$\omega(x) \rightarrow$ lateral deformation

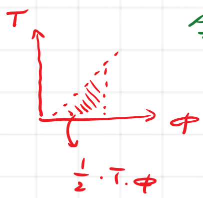
$\phi(x) \Rightarrow$ torsional deformation

$\omega(x)$, $\phi(x)$,

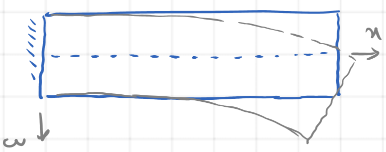
$$\Pi = \frac{1}{2}\int_0^L E \cdot I_z \cdot \omega''(x)^2 \, dx + \frac{1}{2}\int_0^L G \cdot I_t \cdot \phi'^2(x) \, dx + \frac{1}{2}\int_0^L E \cdot I_\omega \cdot \phi''^2(x) \, dx$$

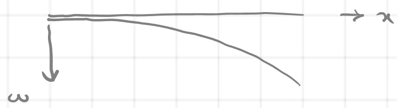
$$- \int_0^L \left(M_y(x) \cdot \phi(x)\right)' \cdot \omega'(x) \, dx + \underbrace{\frac{1}{2} \cdot T \cdot \phi(L)}_{\frac{1}{2} \cdot P \cdot a \cdot \phi^2}$$



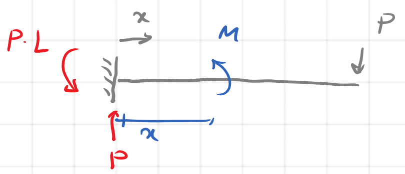


$P \cdot \underbrace{\sin\phi}_{\phi} \cdot a = P \cdot a \cdot \phi$



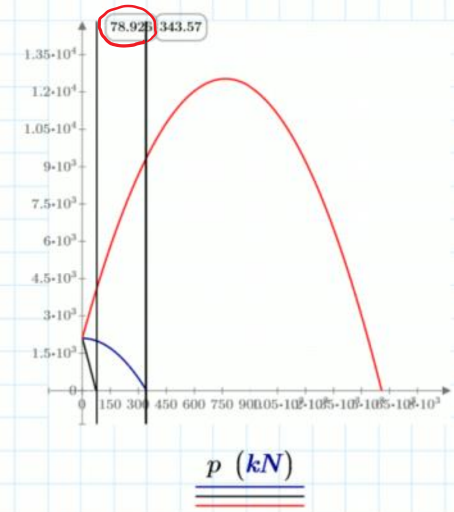


$$\omega(x) = A \cdot \left[1 - \cos\left(\frac{\pi x}{2L}\right)\right]$$



$$\phi(x) = B\left[1 - \cos\left(\frac{\pi x}{2L}\right)\right]$$

$M = P \cdot x - P \cdot L$



stability of structures
Chai H, Yoo, Zell

$Sol(p, 0\ mm)\ (kN^2)$

$Sol(p, -120\ mm)\ (kN^2)$

$Sol(p, 120\ mm)\ (kN^2)$

$$P_{cr} = \frac{4.0126}{\ell^2}\sqrt{EI_y GK_T}$$

$$P_{cr} := \frac{4.0126}{l^2} \cdot \sqrt{E \cdot I_z \cdot G \cdot I_t} = 78.53\ kN$$

$$\gamma_2 := \frac{4.013}{\left(1 - \sqrt{\frac{E \cdot I_w}{G \cdot I_t \cdot l^2}}\right)^2} = 12.465$$

$$P_{cr.T} := \frac{\gamma_2}{l^2} \cdot \sqrt{E \cdot I_z \cdot G \cdot I_t} = 243.95\ kN$$

$$P_{cr} = \gamma_2 \sqrt{EI_y GI_t}/\ell^2 \qquad \gamma_2 = 4.013/[1 - \sqrt{EI_\omega/GI_t\ell^2}]^2$$

Timoshenko (1910)