A plate with a thickness of t, length of L, and height of H is supported by three edges. The supports are assumed to be linear hinges. The plate is under a compressive linear load on its edges in the x direction, as shown in the figure.

a) Consider the plate is not stiffened and determine the critical buckling load $N_{cr.x}$.
b) Assume there are n plates as stiffeners on both sides of the plate with the same plate thickness and width of b on each side extended in the x direction and discrete equal spacing in the y direction. Determine the buckling load $N_{cr.x}$ with n interval stiffeners.

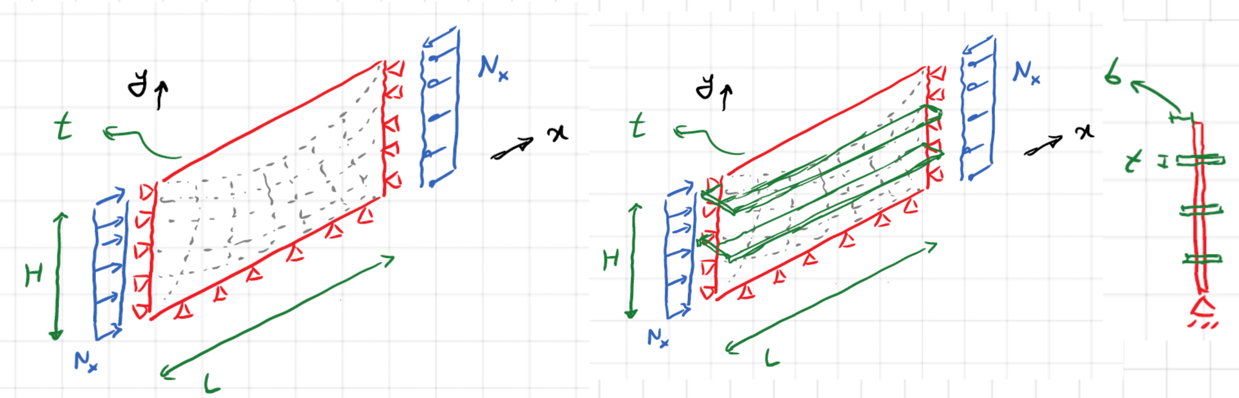
After the parametric solution:

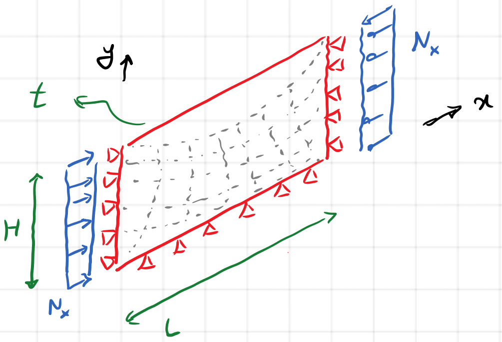
Assume the length and height of the plate are 5m and 3m, respectively. If the material is steel with a modulus of elasticity of $E = 210GPa$ and the Poisson ratio of $v = 0.3$ by a thickness of 5mm plate:

c) What would be the critical buckling load of the plate without any stiffeners?
d) What is the critical buckling load if two stiffeners are used in two rows with the same thickness and 50mm width on each side?

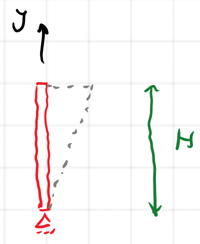
Assume the plate is under a compression force $N_x = 5kN/m$. If the buckling capacity of the plate is expected to be ten times greater than the applied load:

e) Determine the required plate thickness without any stiffener.
f) Determine the required plate thickness using two rows of stiffeners with 50mm width on each side.



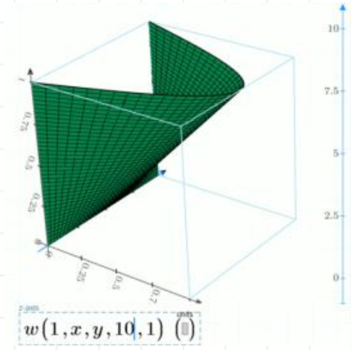


$\sin\left(\frac{\pi x}{L}\right)$



$\frac{y}{H}$

$$w = A \sin\left(\frac{\pi x}{L}\right) \cdot \frac{y}{H}$$



$$\Pi = U + V$$

$$U = \frac{D}{2} \cdot \int_A \left(w_{xx}^{2} + w_{yy}^{2} + 2\nu\, w_{xx} \cdot w_{yy} + 2(1-\nu) \cdot w_{xy}^{2} \right) dA$$

$$V = -\frac{N_x}{2} \int_A w_x^{2}\, dA$$

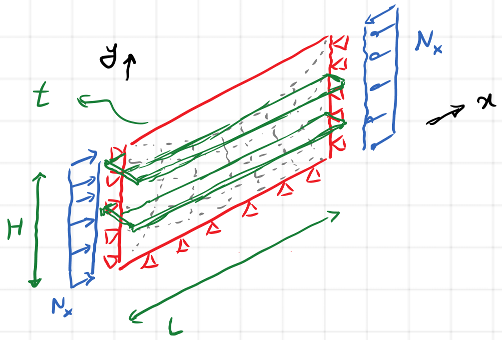
Member or line element $\rightarrow$ $\frac{\partial \Pi}{\partial c}$ = critical load

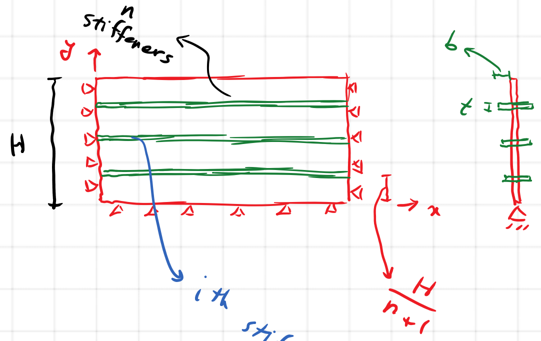
Surface element $\rightarrow$ $\frac{\partial \Pi}{\partial c} = 0 \rightarrow$ equilibrium

$\frac{\partial^2 \Pi}{\partial c^2} = 0 \Rightarrow$ critical load

$$\Pi_{NS}(D,A,L,H,v,N_x) := U_p(D,A,L,H,v) + V_p(N_x,A,L,H)$$

$$N_{x.cr.NS}(D,L,H,v) := \frac{\partial^2}{\partial A^2}\Pi_{NS}(D,A,L,H,v,X) = 0 \xrightarrow{solve,X} \frac{D \cdot H^2 \cdot \pi^2 + (6 \cdot D \cdot L^2 - 6 \cdot D \cdot L^2 \cdot v)}{H^2 \cdot L^2}$$





ith stiffener $\rightarrow$ $y_i = \frac{iH}{n+1}$

$$U_b = \frac{EI}{2} \sum_{y} \int_{A_0}^{L} \omega_{xx}^{2} \, \xcancel{dA} \quad (dx)$$

$$U_b = \frac{EI}{2} \cdot \sum_{y} \int_{0}^{L} \omega_{xx}^{2} \cdot dx$$

$$\omega = A \sin\left(\frac{\pi x}{L}\right) \cdot \frac{y}{H} \rightarrow \omega_{xx} = -A \cdot \frac{\pi^2}{L^2} \cdot \sin\left(\frac{\pi x}{L}\right) \cdot \frac{y}{H}$$

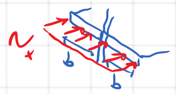
$$U_b = \frac{EI}{2} \cdot \sum_{y} \cdot \int_{0}^{L} A^2 \cdot \frac{\pi^4}{L^4} \cdot \sin^2\left(\frac{\pi x}{L}\right) \cdot \frac{y_i^2}{H^2} \, dx \qquad y_i \rightarrow \left(\frac{iH}{n+1}\right)^2$$

$$U_b = \frac{EI}{2} \cdot A^2 \cdot \frac{\pi^4}{L^4} \cdot \sum_{i=1}^{n} \frac{i^2 \cancel{H}^2}{(n+1)^2 \cdot \cancel{H}^2} \int_{0}^{L} \sin^2\left(\frac{\pi x}{L}\right) \cdot dx$$

$$\left(n = 2 \Rightarrow \sum_{1}^{2} \frac{i^2}{3^2} = \frac{1+4}{9} = \frac{5}{9} \right)$$

$$V = -\frac{P}{2} \int v'^2 \, dx \Rightarrow \qquad P \ (K_N)$$

$$N_x \rightarrow \frac{F}{L} \left(\frac{K_N}{m}\right)$$



$$F = N_x \cdot 2b$$

$$V_b = -\frac{N_x \cdot 2b}{2} \sum \int_{\cancel{A_0}}^{L} \omega_x^2 \, \cancel{dA} \rightarrow dx$$

$$V_b = -\frac{N_x \cdot 2b}{2} \sum \int_{\cancel{A}\,0}^{L} \omega_x^2 \, \cancel{dA} \to dx$$

$$\omega = A \sin\left(\frac{\pi x}{L}\right) \cdot \frac{y}{H}$$

$$\omega_x = A \cdot \frac{\pi}{L} \cdot \cos\left(\frac{\pi x}{L}\right) \cdot \frac{y}{H}$$

$$\to V_b = -\frac{N_x \cdot 2b}{2} \cdot \sum_{i=1}^{n} \int_0^L A^2 \cdot \frac{\pi^2}{L^2} \cdot \cos^2\left(\frac{\pi x}{L}\right) \cdot \frac{y^2}{H^2} \cdot dx$$

$$\Pi_{WS}(D,A,L,H,v,N_x,E,I,b,n) := U_p(D,A,L,H,v) + V_p(N_x,A,L,H) + U_b(A,L,H,E,I,n) + V_b(N_x,A,L,H,b,n)$$

$$N_{x.cr.WS}(D,L,H,v,E,I,b,n) := \frac{\partial^2}{\partial A^2} \Pi_{WS}(D,A,L,H,v,X,E,I,b,n) = 0 \xrightarrow{solve, X}$$

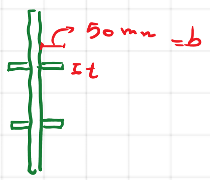
$E = 210$ GPa, $t = 5$ mm, $L = 5$ m, $H = 3$ m, $\nu = 0.3$, $D = \frac{E \cdot t^3}{12(1-\nu^2)}$

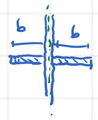
without stiffeners $\to$ $N_{x,cr} \cong 2$ kN/m

$n = 2$

$b = 50$ mm

$t = 5$ m





$$I = \frac{t(2b)^3}{12}$$

with stiffeners $\to$ $N_{x,cr} \cong 20$ kN/m

Assume the given plate is under compressive load in x direction as 5 kN/m. If the buckling capacity is expected to be 10 time greater that applied load:

a) what is the required plate thickness without stiffeners?

b) what is the required plate thickness with 2 rows stiffeners? (assume the plate thickness and stiffeners' thickness are the same)

Guess Values

$t_g := 5\ mm$

Constraints

$N_{x.cr.NS}(D(t_g), L, H, v) = 50\ \frac{kN}{m}$

Solver

$\text{Find}(t_g) = 14.452\ mm$

Guess Values

$t_g := 5\ mm$

Constraints

$N_{x.cr.WS}(D(t_g), L, H, v, E, I(t_g), b, n) = 50\ \frac{kN}{m}$

Solver

$\text{Find}(t_g) = 9.75\ mm$