In the previous video, we delved into the topic of overdamped structures and learned about the concept of critical damping. In this video, we will be focusing on underdamped systems and their behavior. We will examine the structure's response under different conditions and obtain the relevant response equations. By the end of this video, you will better understand how underdamped systems behave and how to calculate their response.

$$r^2 + 2\xi\omega r + \omega^2 = 0$$

$$\rightarrow \quad r = -\xi\omega \pm \omega\sqrt{\xi^2 - 1}$$

$\xi < 1$ (underdamped)

$$r = -\xi\omega \pm \omega\sqrt{(-1)(1-\xi^2)} = \underbrace{-\xi\omega}_{\text{real}} \pm \underbrace{i\omega\sqrt{1-\xi^2}}_{\text{imaginary}}$$

$$u(t) = e^{-\xi\omega t}\left[A\cos(\underbrace{\omega\sqrt{1-\xi^2}}_{\omega_d}\, t) + B\sin(\underbrace{\omega\sqrt{1-\xi^2}}_{\omega_d} \cdot t)\right]$$

$$u(t) = e^{-\xi\omega t}\left[A\cos\omega_d t + B\sin\omega_d t\right]$$

$$\dot{u}(t) = -\xi\omega e^{-\xi\omega t}\left[A\cos\omega_d t + B\sin\omega_d t\right] + e^{-\xi\omega t}\left[-\omega_d A\sin\omega_d t + \omega_d \cdot B\cos\omega_d t\right]$$

$$u(t=0) = u_0 \Rightarrow 1(A + 0) = u_0 \rightarrow \boxed{A = u_0}$$

$$\dot{u}(t=0) = \dot{u}_0 \Rightarrow -\xi\omega(A) + 1(\omega_d B) = \dot{u}_0 \Rightarrow \boxed{B = \frac{\dot{u}_0 + \xi\omega \cdot u_0}{\omega_d}}$$

$$\boxed{u(t) = e^{-\xi\omega t}\left[u_0\cos\omega_d t + \frac{\dot{u}_0 + \xi\omega \cdot u_0}{\omega_d} \cdot \sin\omega_d t\right]}$$