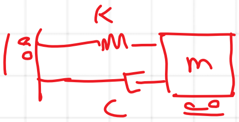
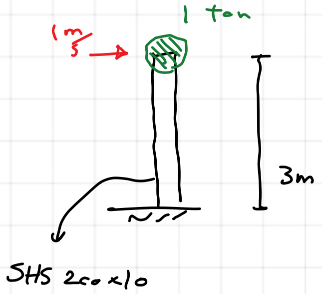
In the previous video, we delved into a detailed discussion of the behavior of underdamped systems. We analyzed the structure's response under different conditions and outlined the corresponding response equations. We explored the concept of damping, which is the resistance of a system to oscillation, and discussed how underdamped systems oscillate with a decreasing amplitude over time. This video aims to provide a better understanding of the subject by presenting two examples that will help to clarify the concepts discussed in the previous video.

$$u(t) = e^{-\xi\omega t}\left[u_0 \cos\omega_d t + \frac{\dot{u}_0 + \xi\omega \cdot u_0}{\omega_d} \cdot \sin\omega_d t\right]$$



$$\omega = \sqrt{\frac{k}{m}}, \quad \omega_d = \omega\sqrt{1-\xi^2}$$



$u_0 = 0$

$\dot{u}_0 = 1 \ \frac{m}{s}$

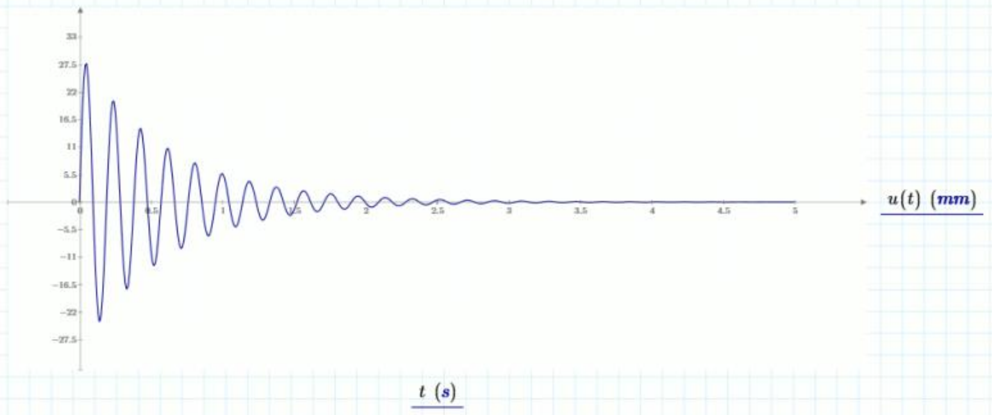
$k = \frac{3EI}{h^3}$, $M = 1$ ton

$u_0 := 0\ m$ $\quad u'_0 := 1\ \frac{m}{s}$ $\quad I := 44.71 \cdot 10^6 \cdot mm^4$ $\quad h := 3\ m$ $\quad E := 200\ GPa$ $\quad M := 1\ ton$

$k := \frac{3 \cdot E \cdot I}{h^3} = 993.556\ \frac{kN}{m}$ $\quad w := \sqrt{\frac{k}{M}} = 33.094\ \frac{rad}{s}$ $\quad T := \frac{2 \cdot \pi}{w} = 0.19\ s$ $\quad f := \frac{1}{T} = 5.267\ Hz$

$\xi := 0.05$ $\quad w_d := w \cdot \sqrt{1-\xi^2} = 33.053\ \frac{1}{s}$

$u(t) := e^{-\xi \cdot w \cdot t} \cdot \left(u_0 \cdot \cos(w_d \cdot t) + \frac{u'_0 + \xi \cdot w \cdot u_0}{w_d} \cdot \sin(w_d \cdot t)\right)$ $\quad t := 0, 0.01 .. 5\ s$



$F = k \cdot \delta$

$\downarrow$

$F_{max} = k\ \delta_{max}$

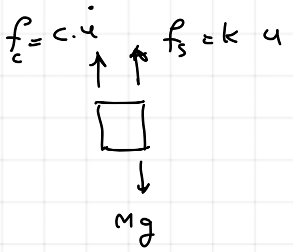
$F_{max} = 993.6\ \frac{kN}{m} \times 28\ mm$

$F_{max} = 28\ kN$

$M_{max} = F_{max}\ h$

$M_{max} = 28\ kN \times 3\ m$

$\boxed{M_{max} = 83.5\ kN\ m}$



$m\ddot{u} + c\dot{u} + ku = mg$

$\ddot{u} + \underbrace{\frac{c}{m}}_{2\xi\omega}\dot{u} + \underbrace{\frac{k}{m}}_{\omega^2}u = g$ $\qquad \xi < 1$

$u_h(t) = e^{-\xi\omega t}(A \cos\omega_d t + B \sin\omega_d t)$

$u_p(t) = \frac{g}{k/m} = \frac{mg}{k}$

$$\boxed{u(t) = e^{-\xi\omega t}(A \cos\omega_d t + B \sin\omega_d t) + \frac{mg}{k}}$$

$$u(t) = e^{-\xi \omega t}\left(A\cos\omega_d t + B\sin\omega_d t\right) + \frac{mg}{K}$$

$$\dot{u}(t) = -\xi\omega e^{-\xi\omega t}\left(A\cos\omega_d t + B\sin\omega_d t\right)$$

$$+ e^{-\xi\omega t}\left(-\omega_d A\sin\omega_d t + \omega_d B\cos\omega_d t\right)$$

$$u_0 = 0 \Rightarrow 0 = 1(A) + mg/K \Rightarrow \boxed{A = \frac{-mg}{K}}$$

$$\dot{u}_0 = 0$$

$$\Rightarrow 0 = -\xi\omega(A) + (\omega_d \cdot B) \Rightarrow B = \frac{A\,\xi\,\omega}{\omega_d}$$

$$\rightarrow \boxed{B = \frac{-mg/K\ \omega \cdot \xi}{\omega_d}}$$

$$\frac{k}{m} = \omega^2$$

$$\frac{m}{K} = \frac{1}{\omega^2}$$

$$\boxed{B = -\frac{g \cdot \xi}{\omega \cdot \omega_d}}$$

$$u(t) = e^{-\xi\omega t}\left[-\frac{mg}{K}\cos\omega_d t - \frac{g\,\xi}{\omega\,\omega_d}\cdot\sin\omega_d t\right] + \frac{mg}{K}$$

$E := 200\ GPa \qquad I := 36.92 \cdot 10^{6} \cdot mm^{4} \qquad l := 2\ m \qquad M := 1\ ton$

$k := \frac{3 \cdot E \cdot I}{l^{3}} = \left(2.769 \cdot 10^{3}\right) \frac{kN}{m} \qquad w := \sqrt{\frac{k}{M}} = 55.248\ \frac{rad}{s} \qquad T := \frac{2 \cdot \pi}{w} = 0.114\ s \qquad f := \frac{1}{T} = 8.793\ Hz$

$$w_d(\xi) := w \cdot \sqrt{1-\xi^2}$$

$$u(t,\xi) := e^{-\xi \cdot w \cdot t} \cdot \left(\frac{-M \cdot g}{k} \cdot \cos\left(w_d(\xi) \cdot t\right) - \frac{g \cdot \xi}{w \cdot w_d(\xi)} \cdot \sin\left(w_d(\xi) \cdot t\right) \right) + \frac{M \cdot g}{k} \qquad t := 0, 0.01\ s..2\ s$$